# Joint optimization of distortion and cut location for mesh parameterization using an Ambrosio-Tortorelli functional

Colin Weill-Duflos, David Coeurjolly, Jacques-Olivier Lachaud and Fernando de Goes

# Outline

# Outline

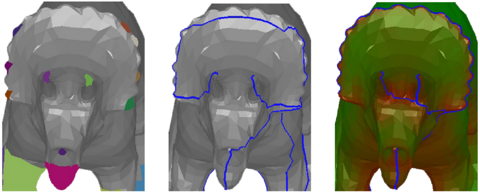
# Introduction

## Usual workflow

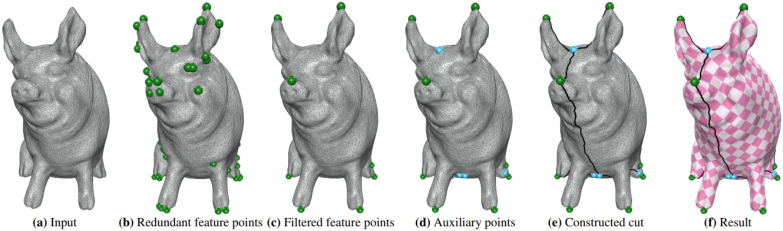
- Decompose into topological disks
- Compute first mapping for each disks
- Minimize disks distorsion energy (ARAP, Sym Dirichlet, MIPS...)

# Introduction

## Related works : cutting methods



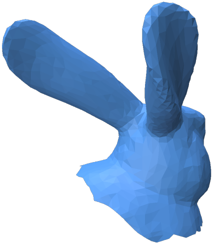
Seamster [Sheffer and Hart. 2002]



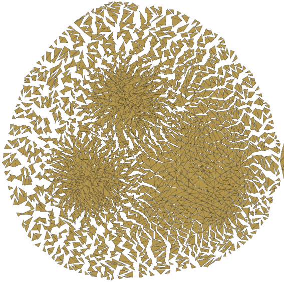

Greedy Cut [Zhu et al. 2020]

# Introduction
## Problem

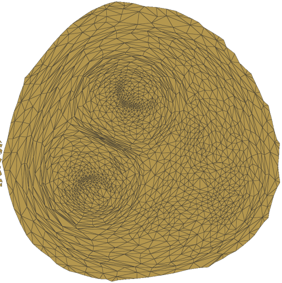
- Cut has to predict distortion
- Balance between cut length and distortion



Domain



Full cut, no distortion



No cut, high distortion

# Introduction

## Our contribution

Joint optimization of distorsion and cuts

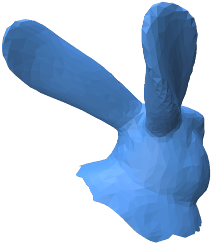
Related works:

- Autocuts: simultaneous distortion and cut optimization for UV mapping (Roi Poranne, Marco Tarini, Sandro Huber, Daniele Panozzo, and Olga Sorkine-Hornung, 2017)
- Optcuts: joint optimization of surface cuts and parameterization (Minchen Li, Danny M. Kaufman and Vladimir G. Kim and Justin Solomon and Alla Sheffer, 2018)

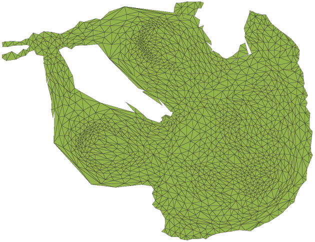
# Outline

# Ambrosio-Tortorelli functional

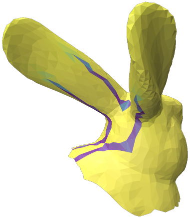
## Our goal



Domain



$u$: parameterization function



$v$: cuts indicator

Our goal is to build a variational approach using two functions, $u$ representing the parameterization and $v$ representing the cuts

# Ambrosio-Tortorelli functional

## The Mumford-Shah functional

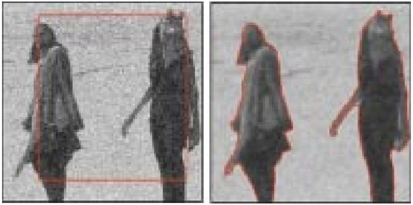
Segmentation as a piecewise-smooth function

$$\mathscr{M}\mathscr{S}(K,u) = \underbrace{\alpha \int_{M\setminus K} |u-g|^2 dx}_{\text{Fitting term}} + \underbrace{\int_{M\setminus K} |\nabla u|^2 dx}_{\text{Smoothing term}} + \underbrace{\lambda \mathscr{H}^1(K \cap M)}_{\text{Discontinuities term}} \tag{1}$$

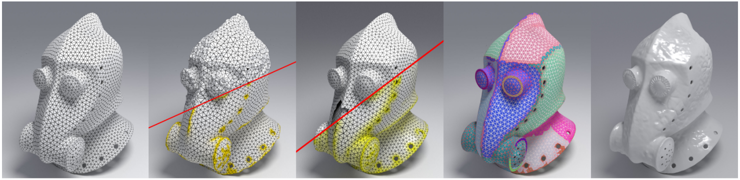
- $M$ the object surface
- $g$ the input data, defined on $M$
- $u$ the regularized data
- $K$ the set of discontinuities
- the $\mathscr{H}^1$ Hausdorff measure
- $\alpha$ et $\lambda$ two real numbers

# Ambrosio-Tortorelli functional

## Mumford-Shah related Prior Works



[Tsai et al. 2001]



(a) Original (b) Denoising (c) Inpainting (d) Segmentation (e) Embossing

[Bonneel et al. 2018]

# Ambrosio-Tortorelli

## The Ambrosio Tortorelli functional

Mumford-Shah relaxation

$$\mathscr{A}\mathscr{T}(u,v) = \underbrace{\alpha \int_M |u-g|^2 dx}_{\text{Fitting term}} + \underbrace{\int_M |v \nabla u|^2 dx}_{\text{Smoothing term}} + \underbrace{\lambda \int_M \varepsilon (\nabla v)^2 + \frac{1}{4\varepsilon}(1-v)^2 dx}_{\text{Discontinuities term}} \quad (2)$$

With $v$: $M \to [0,1]$, and $\varepsilon$ a real positive number.

# Energy formulation

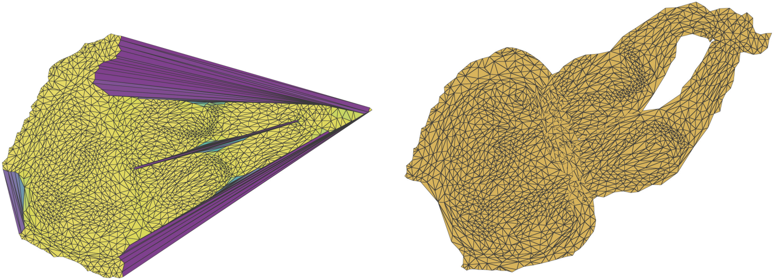
$$\mathscr{A}\mathscr{T}(u,v) = \underbrace{\alpha \int_M |u-g|^2 dx}_{\text{No use for us}} + \underbrace{\int_M |v\nabla u|^2 dx}_{\text{Weighted Dirichlet}}$$
$$+ \lambda \int_M \varepsilon(\nabla v)^2 + \frac{1}{4\varepsilon}(1-v)^2 dx$$

Per face energy :

$$\begin{aligned} AT_{modif}(\mathrm{u},v) &= (v^2+\gamma)\left((\nabla \mathrm{u})^2 + (\nabla \mathrm{u}^{-1})^2\right) \quad && + \lambda\varepsilon(\nabla v)^2 + \frac{\lambda}{4\varepsilon}(1-v)^2 \\ &= (v^2+\gamma)\Psi(u) && + \lambda\varepsilon(\nabla v)^2 + \frac{\lambda}{4\varepsilon}(1-v)^2 \end{aligned}$$

Where $\gamma << 1$ is a constant (for stability) $u$ is placed on vertices, $v$ on faces

# Method outline



The color represents $v$ on each face, yellow means the face is rigid ($v = 1$) and purple means the face can be distorted ($v = 0$).
We first optimize $v$ and $u$ (left), then we compute cuts along $v$ and finally optimize $u$ to obtain the final parameterization (right)

# Method outline

## Optimization method

In AT, with $\varepsilon$ fixed : $u$ (resp. $v$) fixed -> convex quadratic expression in $v$ (resp. $u$)

$\varepsilon \leftarrow \varepsilon_1$
**for** $\varepsilon \geq \varepsilon_2$ **do**
    **loop**
        Fix $v$, solve $u$
        Fix $u$, solve $v$
    **end loop**
    Decrease $\varepsilon$
**end for**

In our case, the 'Fix $v$ solve $u$' step means optimizing a Symmetric Dirichlet energy, and consists of most of the time spent by our method. We use the method from "Analytic eigensystems for isotropic distortion energies" [Smith et al. 2019]

# Method outline

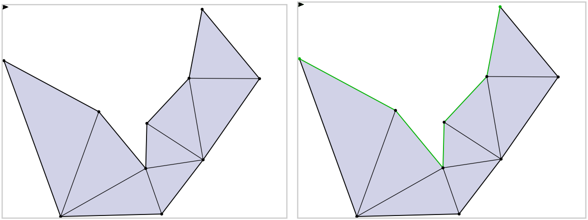
## Cutting method

Cuts must be defined from face patches

Ill posed problem: similar to homotopy reduction, but not exactly

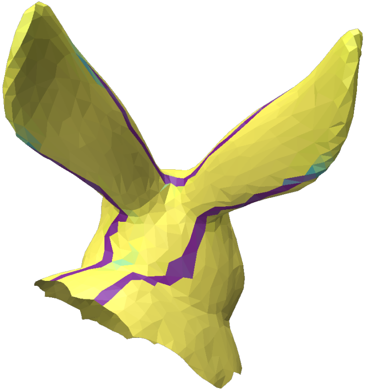
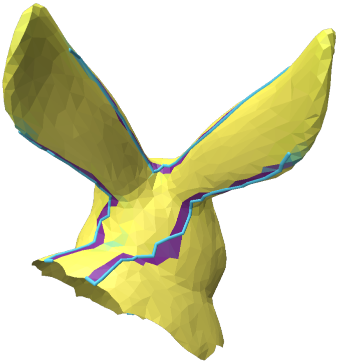
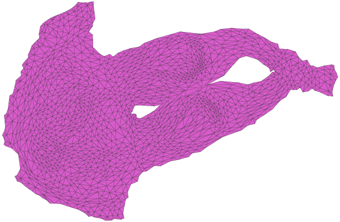
Retained method:

- Shortest pair between farthest points

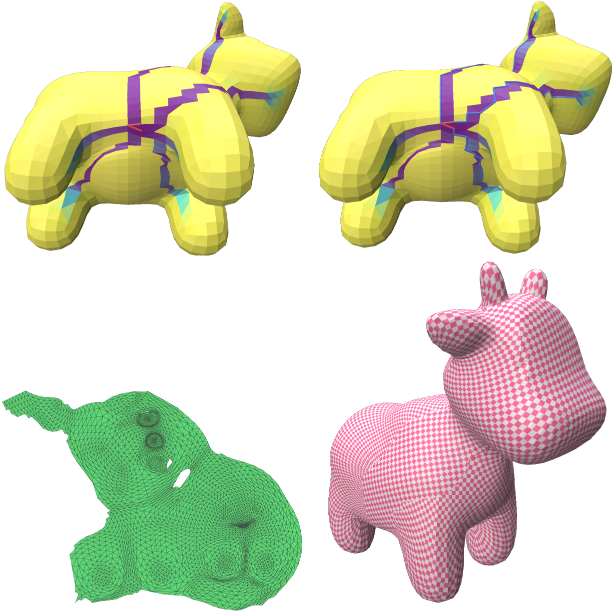


# Outline

# Results

# Results



# Results

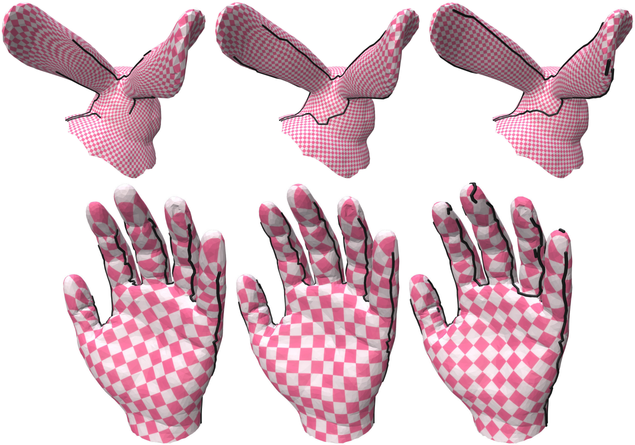
## Comparison to existing method (OptCuts)

| Model | Bunny head | Camel head | bimba | armadillo |
|---|---|---|---|---|
| OptCut's time | 2.5s | 89s | 13s | 29s |
| Our time | **1.6s** | **35s** | **6s** | **6.1s** |
| Initial distorsion | 9.10 | 7.45 | 7.64 | 10.3 |
| OptCut's distorsion | **4.36** | 4.26 | 4.50 | 5.22 |
| Our distorsion | 4.39 | 4.26 | 4.50 | 5.22 |
| OptCut's cut length | **2.88** | 2.93 | 2.01 | **2.07** |
| Our cut length | 3.82 | **2.56** | **1.90** | 4.25 |

Table: We achieve results with similar quality as OptCuts but in less time

# Parameters

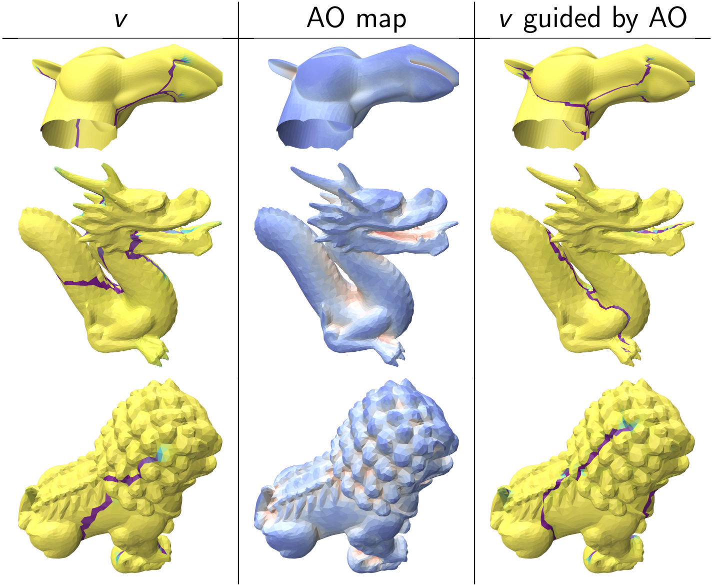
## Parameters



Results on the `bunnyhead` and `hand` meshes for $\lambda = \{3, 1, 0.1\}$. Lower $\lambda$ parameter implies longer cuts.

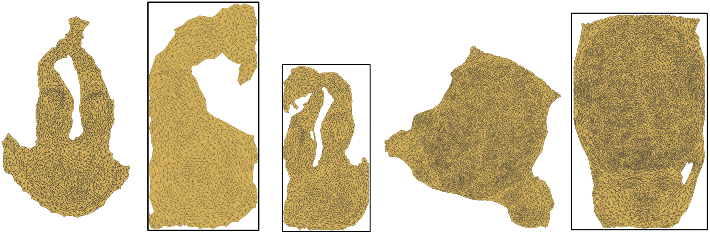
# Constraints

## Ambient Occlusion map



# Constraints

## Texture packing



(a) (b) (c) (d) (e)

# Conclusions and future works

Future works:

- Test other energies such as ARAP, Symmetric ARAP, MIPS...
- Use a cutting method following the topology induced by $v$
- Speed up optimization using recent competitive gradient and mirror descent algorithms [Schäfer and Anandkumar, 2019; Schäfer et al. 2020]

# Questions?