Tangent estimation along 3D digital curves

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Abstract

In this paper, we present a new three-dimensional (3D) tangent estimator by extending the well-known two-dimensional (2D) λ-maximal segment tangent (λ-MST) estimator, which has very good theoretical and practical behaviors. We show that our proposed estimator keeps the same time complexity, accuracy and experimental asymptotic behaviors as the original 2D one.

1. Introduction

The accurate estimation of geometric parameters such as tangent directions along a digital curve is an essential step in many applications. In medical images, 3D digital curves are often obtained as the results of 3D curvilinear skeletonisation process [2], and play a major role in numerous purposes. For instance, in quantitative analysis of human airway trees based on CT images, a tangent direction at each point of the skeleton of the tree is used to define accurately its cross section [11] which allows to make reliable measurements. Different tangent estimators have been already studied in the literature. In the framework of digital geometry, there exist few studies on 3D digital curves yet while there are numerous methods performed on 2D digital curves such that each tangent is evaluated using a finite set of curve points around the point of interest. The size of such a finite set of neighboring points is either fixed globally by users [10, 12] or adopted by the local shape geometry obtained by recognizing digital straight line segments around the point of interest [8, 4]. Note that most of the 2D methods do not work straightforwardly with 3D digital curves, and they often have unexpected behavior for some special configurations of curve points.

In this paper, we present a new 3D tangent estimator which is an extension of the algorithm presented by Lachaud et al. in [9], called λ-MST, and originally designed for estimating tangents on 2D digital contours. It is a simple method based on maximal straight segments recognition [8] along digital contour, and has very good properties such as linear computation complexity and accurate results. Moreover, it has multigrid convergence proven in [9]. The main contribution of this paper is to present a 3D version of this method that maintains the same time complexity, accuracy and asymptotic behavior; the latest is shown experimentally on several space parametric curves.

2. Basic notions

In this section, we recall some basic notions and properties related to 3D digital curves, which are necessary to understand the sequel of the paper. More extensive reviews are provided in [1, 5, 9].

Let $\mathbb{Z}$ be the set of integers, $\mathbb{Z}_*$ the set of non-negative integers, and $\mathbb{R}_+$ the set of strictly positive real values. We denote by $\mathbb{Z}^n$, where $n \in \mathbb{Z}$, the discrete grid of dimension n. In this paper, we focus on 2D and 3D cases, thus $n = 2$ or 3. Any point of $\mathbb{Z}^n$ is defined by an n-tuple $(x_1, ..., x_n)$ with $x_i \in \mathbb{Z}$, representing the coordinates in the discrete space. The number of points in $X \subset \mathbb{Z}^n$, *i.e.* the *cardinality* of X, is denoted by $|X|$. Let $x = (x_1, ..., x_n)$ and $y = (y_1, ..., y_n)$ for $x, y \in \mathbb{Z}^n$. Then, the function $d^2 : \mathbb{Z}^n \times \mathbb{Z}^n \to \mathbb{Z}$, which is the *squared Euclidean distance* between x and y, is defined by $d^2(x, y) = \sum_{i=1}^{n}(x_i - y_i)^2$. Given a point $x \in \mathbb{Z}^3$, the three adjacency sets are defined:

$$\Gamma_6(x) = \{y \in \mathbb{Z}^3 \mid d^2(x, y) \leq 1\}, \quad (1)$$
$$\Gamma_{18}(x) = \{y \in \mathbb{Z}^3 \mid d^2(x, y) \leq 2\}, \quad (2)$$
$$\Gamma_{26}(x) = \{y \in \mathbb{Z}^3 \mid d^2(x, y) \leq 3\}, \quad (3)$$

so that a point y is a *k-neighbor* of x if $y \in \Gamma_k(x)$.

We may now define 3D digital curves [1].

Definition 1 *Any set $C \subset \mathbb{Z}^3$ such that $|C| \geq 2$ is called a 3D digital k-curve, or simply called k-curve, iff there are exactly two points $p, q \in C$ such that $|\Gamma_k(p) \cap C| = |\Gamma_k(q) \cap C| = 2$ and for any $x \in C \setminus \{p, q\}$, we have $|\Gamma_k(x) \cap C| = 3$.*

Our tangent estimator on a k-curve relies on the following arithmetical definitions of digital lines [1, 5]. We first give the definition of 2D digital line as any 3D digital line is constructed from two projected 2D digital lines.

Definition 2 *A 2D digital line with direction vector $(b, a) \in \mathbb{Z}^2$, shift μ and thickness e, where $a, \mu \in \mathbb{Z}$ and $b, e \in \mathbb{Z}_*$ such that $\gcd(a, b) = 1$, is defined as the set of points $(x, y) \in \mathbb{Z}^2$ which satisfy the diophantine inequality:*

$$\mu \leq ax - by < \mu + e,$$

and denoted by $\mathcal{D}_2(a, b, \mu, e)$.

Definition 3 *A 3D digital line with main vector $(a, b, c) \in \mathbb{Z}^3$ such that $|a| \geq |b| \geq c$, shifts μ, μ', and thicknesses e, e', where $a, b, \mu, \mu' \in \mathbb{Z}$, $c, e, e' \in \mathbb{Z}_*$, is defined as the set of points $(x, y, z) \in \mathbb{Z}^3$ which satisfy the diophantine inequalities:*

$$\begin{aligned} \mu &\leq cx - az < \mu + e, \quad (4) \\ \mu' &\leq bx - ay < \mu' + e', \quad (5) \end{aligned}$$

and denoted by $\mathcal{D}_3(a, b, c, \mu, \mu', e, e')$.

The definition for coefficients ordered in different way from $|a| \geq |b| \geq c$ may be obtained by permuting x, y, z as well as their coefficients.

It should be mentioned that the thicknesses e and e' control the adjacency relation of a 3D digital line [1]. Hereafter we use one of the following three settings; a 3D digital line $\mathcal{D}_3(a, b, c, \mu, \mu', e, e')$ is:

- a 6-curve if we set $e = |a| + c$ and $e' = |a| + |b|$,
- a 18-curve if we set either $e = |a| + c$ and $e' = |a|$, or $e = |a|$ and $e' = |a| + |b|$,
- a 26-curve if we set $e = e' = |a|$.

From Definition 1, we easily see that any k-curve C has a totally ordered point set $(x_1, x_2, \ldots, x_{|C|})$ for all $x_i \in C$, so that we can define a set of consecutive points of C from the i-th point to the j-th point, denoted by $C_{i,j}$. With this definition of a part of a k-curve, we can define the following two notions, digital straight segment and maximal segment, which are originally defined for 2D [5, 8, 9] and can be extended to 3D.

Definition 4 *Given a k-curve C, a set of its consecutive points $C_{i,j}$ where $1 \leq i \leq j \leq |C|$ is said to be a digital straight segment (or $S(i, j)$) iff there exists a digital line $\mathcal{D}_3$ containing all the points of $C_{i,j}$.*

The next property [1] is led by Definitions 3 and 4.

Property 1 *$S(i, j)$ is verified iff two of the three projections of $C_{i,j}$ on the basic planes O_{XY}, O_{XZ} and O_{YZ} are 2D straight line segments.*

Definition 5 *Any subset $C_{i,j}$ of C is called a maximal segment iff $S(i, j)$ and $\neg S(i, j + 1)$ and $\neg S(i - 1, j)$.*

The following property is derived by using the notion of the maximality of saturated set presented in [7].

Property 2 *For any k-curve C, there is a unique set $\mathcal{M}$ of its maximal segments, called the tangential cover.*

3. 3D tangential cover construction

The main part of the λ-MST estimator consists in obtaining the tangential cover $\mathcal{M}$ of a given k-curve C, similarly to the original 2D estimator [9]. Thanks to Property 1, we give Algorithm 1, which finds the set of maximal segments by recognizing two 2D digital straight segments of the projections of C. Figure 1 illustrates an example of C and its tangential cover $\mathcal{M}$.

Algorithm 1 Tangential Cover (**Input** C; **Output** $\mathcal{M}$)

$\mathcal{M} \leftarrow \emptyset$
$\mathcal{N} \leftarrow$ empty queue
$\mathcal{E} \leftarrow$ NULL
foreach point $p_i \in C$ **do**
    $M_i \leftarrow \emptyset$
    Push_element($M_i, \mathcal{N}$)
    foreach element M_j in $\mathcal{N}$ **do**
        $s \leftarrow 0$
        If isDSS_XY($M_j \cup \{p_i\}$) **then** $s \leftarrow s + 1$ **end**
        If isDSS_XZ($M_j \cup \{p_i\}$) **then** $s \leftarrow s + 1$ **end**
        If isDSS_YZ($M_j \cup \{p_i\}$) **then** $s \leftarrow s + 1$ **end**
        If $s \geq 2$ **then** $M_j \leftarrow M_j \cup \{p_i\}$ **end**
        else If $s < 2$ **then**
            If $\mathcal{E}(i)$ is NULL **then**
                $\mathcal{E}(i) \leftarrow j$
                $\mathcal{M} \leftarrow \mathcal{M} \cup M_j$
            end
            Remove_first_element($\mathcal{N}$)
        end
    end
return $\mathcal{M}$

Remark: In Algorithm 1 we denote by $\mathcal{E}(i)$ the table which stores the first point of each maximal segment ending at the i-th point.

Algorithm 1 performs two loops. The first one starts from line 04 and makes $|C|$ iterations. The next loop, nested in the first one, starts from line 07 and in worst case performs $|\mathcal{N}|$ iterations, where $\mathcal{N}$ is a set of all maximal segments covering an actual point. The size of $\mathcal{N}$ is bounded by a finite integer value; in fact $|\mathcal{N}| \leq 22$ on average in 2D [3]. Thus, the presented algorithm is linear in number of points of the curve C if the incremental procedure "isDSS" in steps from 9 to 11 has a constant time complexity. Indeed, for this procedure, we can use one of the efficient methods for 2D digital straight line recognition in constant complexity, for example used in [6], after projecting the current set of points on the planes O_{XY}, O_{XZ}, O_{YZ}.

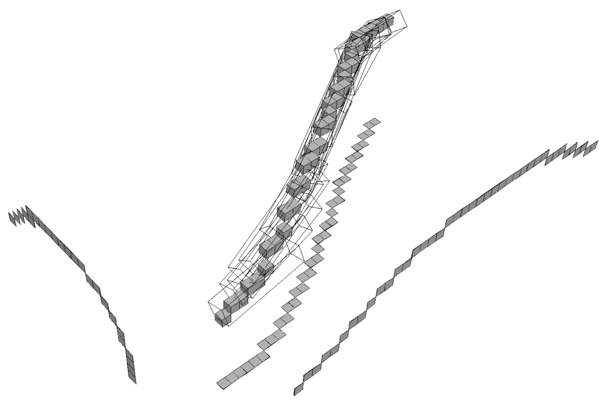

Figure 1. A 3D digital curve and its tangential cover calculated by Algorithm 1.

4. 3D λ-MST estimator

Similarly to the original 2D estimator [9], the 3D estimator at a point x of a k-curve C should depend on the set of all maximal segments going through p. Let us number all the maximal segments of the tangential cover $\mathcal{M}$ of C by increasing indices such that $M_i \in \mathcal{M}$ for $i = 1, 2, \ldots$. Then, such a set is defined by $P(x) = \{M_i \in \mathcal{M}, x \in M_i\}$, called the *pencil of maximal segments* around x. As any point x of a k-curve C is covered by at least one maximal segment, we have the next property similarly to the 2D one [7].

Property 3 *The pencil of maximal segments $P(x)$ of any point x of a k-curve C is never empty.*

In addition, as noted in [8] for 2D cases, several successive points may have the same pencil, and this is also observed for 3D cases. Therefore, the tangent estimator should take also into account the position of the point p within the pencil. More specifically, each point p has the eccentricity with respect to each maximal segment. Let us denote by $L_i = \|n_i - m_i\|_1$ the length of each $M_i = C_{m_i,n_i} \subset C$. Then, the *eccentricity* $e_i(x)$ of a point x with respect to a maximal segment M_i is its relative position between the extremities of M_i such that

$$e_i(x) = \begin{cases} \frac{x-m_i}{L_i} & \text{if } M_i \in P(x), \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The tangent direction is thus estimated by a combination of the directions of maximal segments weighted by a function of the corresponding eccentricity. The function λ maps from [0, 1] to $\mathbb{R}_+$ with $\lambda(0) = \lambda(1) = 0$ and $\lambda > 0$ elsewhere. In this paper, for example, a C^2 function $64(-x^6 + 3x^5 - 3x^4 + x^3)$ is used.

Definition 6 *The 3D λ-MST direction $\boldsymbol{t}(x)$ at point x of a k-curve C is defined as a weighted combination of the vectors $\boldsymbol{t_i}$ of the covering maximal segments M_i such that*

$$\boldsymbol{t}(x) = \frac{\sum_{M_i \in P(x)} \lambda(e_i(x)) \frac{\boldsymbol{t_i}}{|\boldsymbol{t_i}|}}{\sum_{M_i \in P(x)} \lambda(e_i(x))}. \tag{7}$$

From Property 3 and the nature of the eccentricity, this value is always defined and computed with a linear time complexity.

5. Experimental validation

To evaluate the multigrid convergence behavior of the 3D λ-MST estimator, we experiment it on two families of 3D shapes (see Figure 2(a,b)). We measure the error between the expected theoretical tangent and the estimated one with increasing resolutions. On Figure 2(c,d), we can see how the root mean square error (RMSE) evolves. On Figure 2(e), expected and estimated tangents to the helix in a fixed resolution along its principal x axis are shown. Different kinds of error such as maximal error and standard deviation were also evaluated. Results for the helix curve are presented in the table below.

	Helix			
Resolution	10	30	70	100
Std.Dev($\|X - Y\|$)	0.04	0.02	0.007	0.006
Max($\|X - Y\|$)	0.2	0.13	0.079	0.047

6. Conclusions

We have proposed a new tangent estimator for 3D digital curves which is an extension of the 2D λ-MST

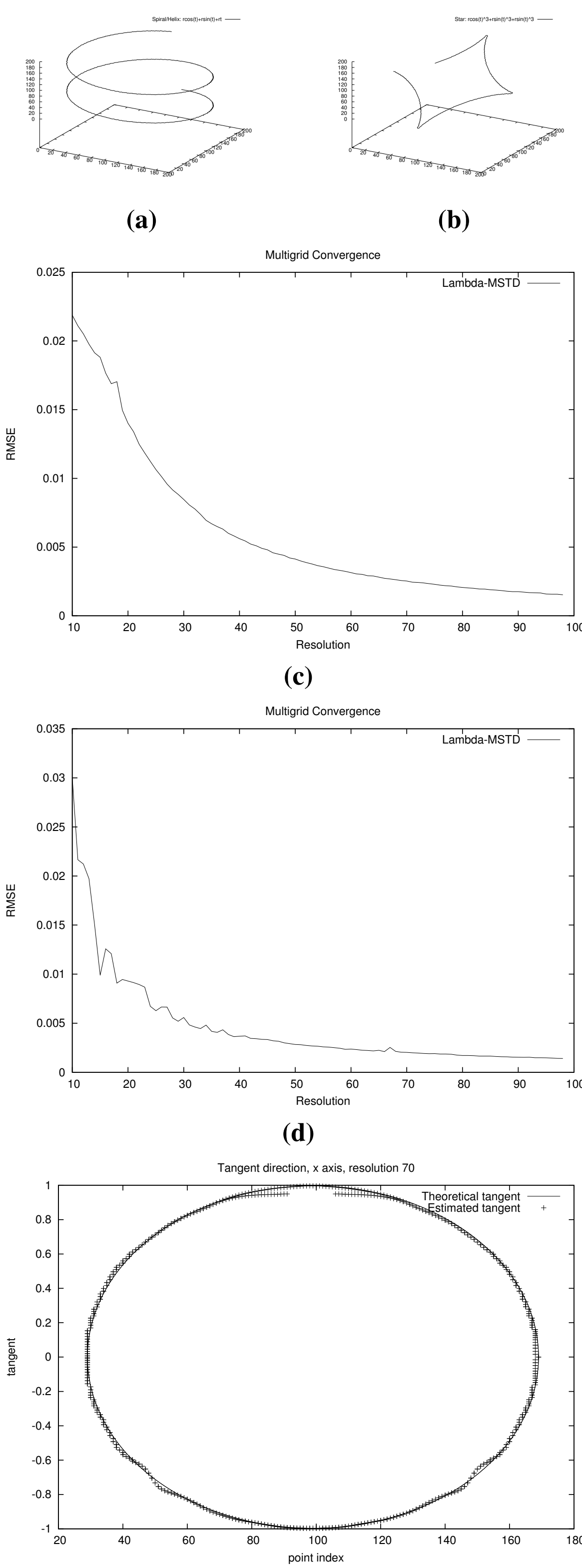

Figure 2. (a,b): two 3D parametric curves (Helix and Star). (c,d): RMSE evolution of (a,b). (e): theoretical and estimated tangents of (a)

estimator. The obtained results show that in our 3D λ-MST algorithm, we keep the same time complexity and accuracy as the original algorithm. Its asymptotic behavior evaluated experimentally on several space parametric curves is promising, while it needs a theoretical proof for 3D case that we work on currently.

The presented algorithm is planed to be used in a practical application such as a module of the system for quantitative human airway tree analysis [11].

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