Meaningful Thickness Detection on Polygonal Curve

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1. Introduction

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- Geometric estimator: best scale to analyze discrete shape.
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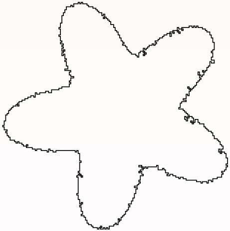
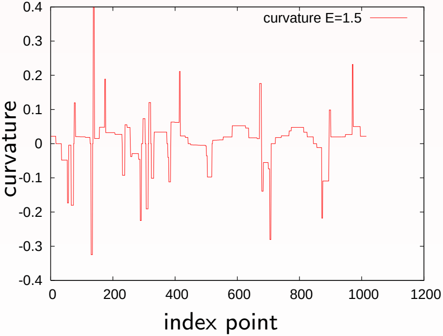
curvature estimator, scale: E=1.5





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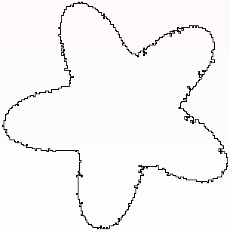
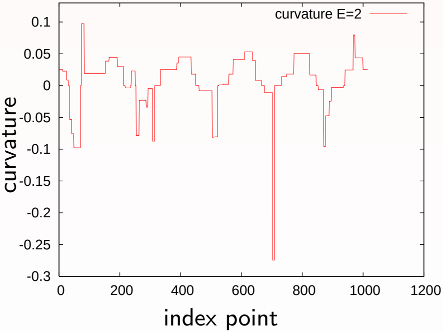


curvature estimator, scale: E=2



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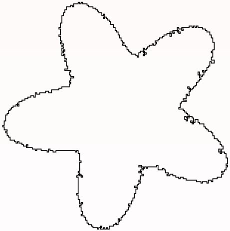
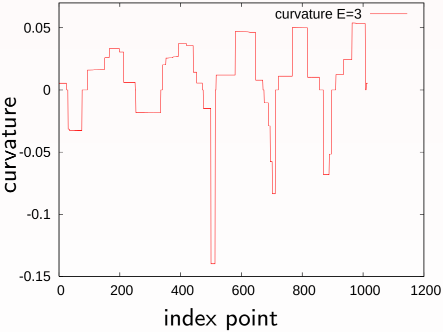


curvature estimator, scale: E=3



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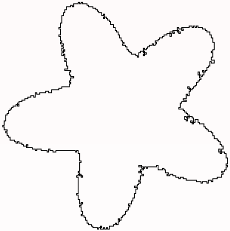
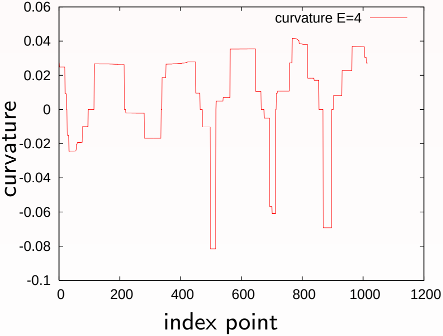
curvature estimator, scale: E=4





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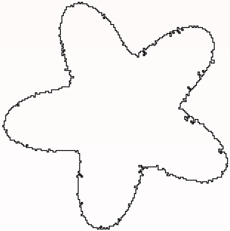
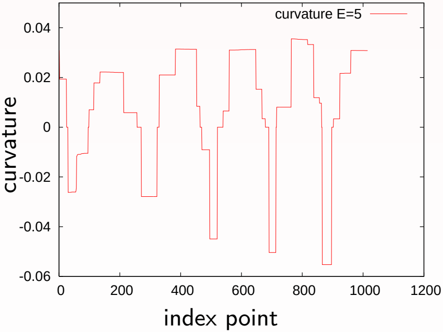
curvature estimator, scale: E=1.5





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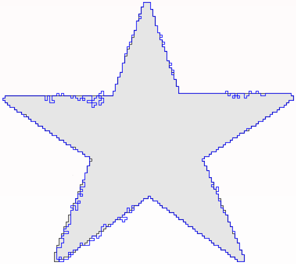
Denoising approach [Hoang et al.,2011] (fidelity parameter $\epsilon = 10$)



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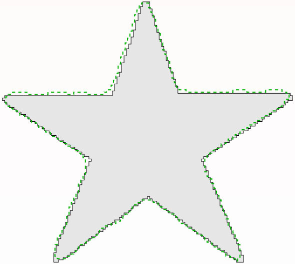

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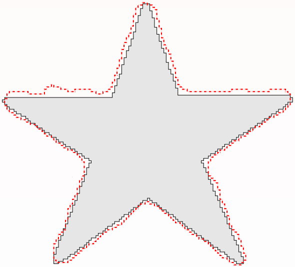

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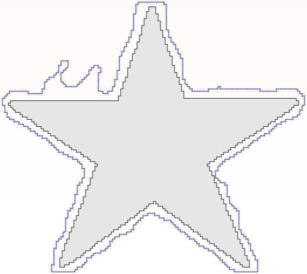

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  - based on perception principle from the Gestalt theory.
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1. A standard Digital Straight Line (DSL):

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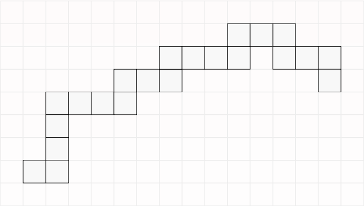

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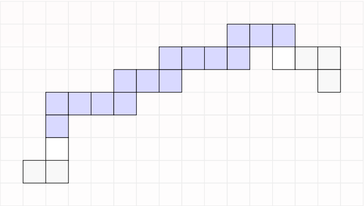

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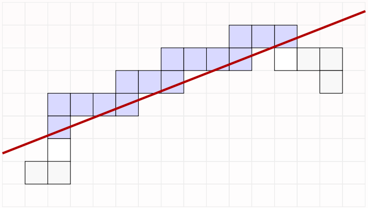

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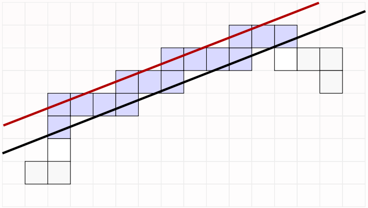

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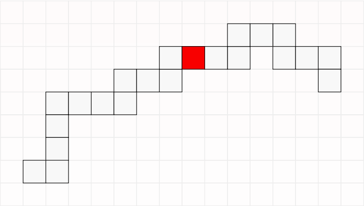

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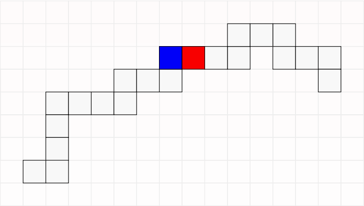

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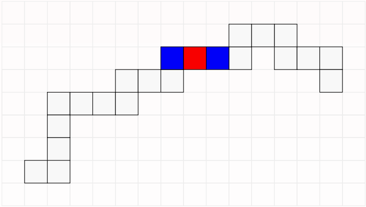

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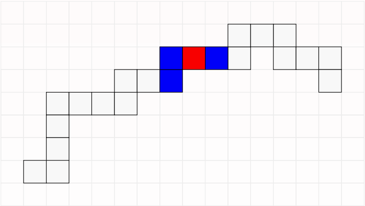

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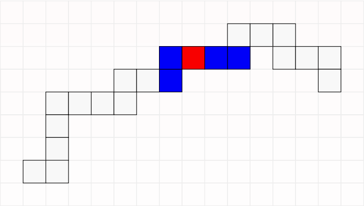

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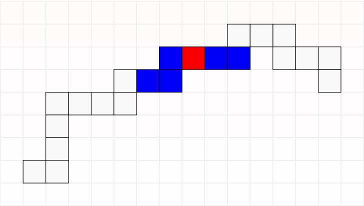

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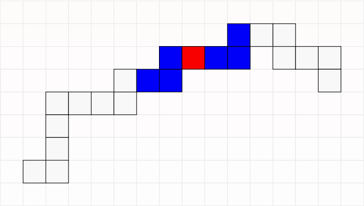

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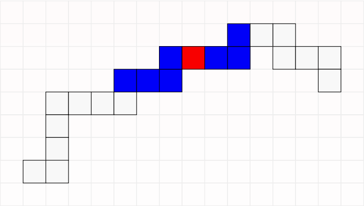

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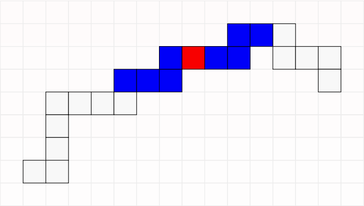

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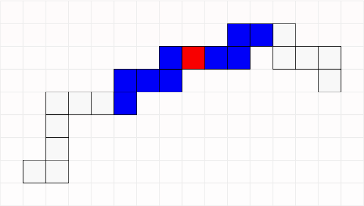

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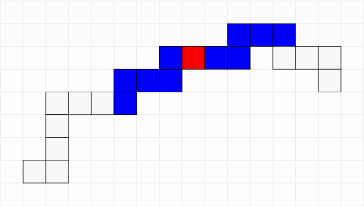

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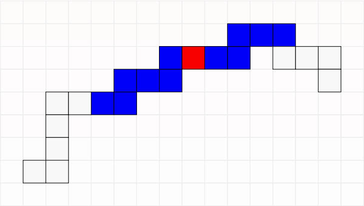

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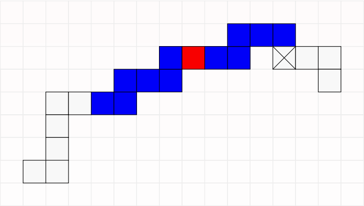

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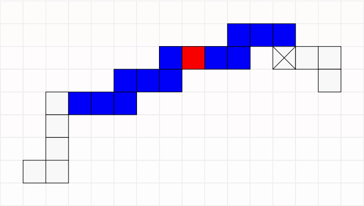

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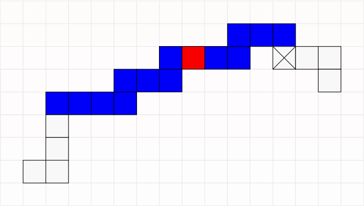

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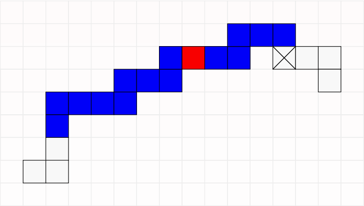

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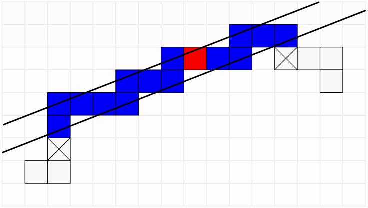

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   ⇒ Length (L) of maximal straight segments
2. They grow longer as h gets finer.
3. Estimate them from a multiresolution decomposition of the input shape.

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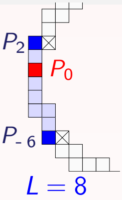

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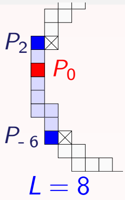
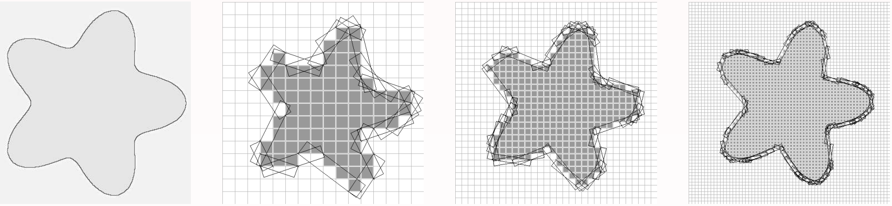




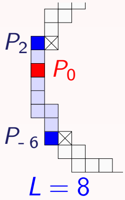
X | $\mathrm{Dig}_2(X)$ | $\mathrm{Dig}_1(X)$ | $\mathrm{Dig}_{0,5}(X)$

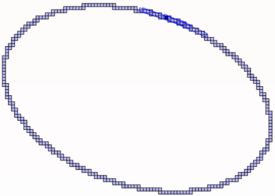
- X some simply connected compact shape of $\mathbb{R}^2$.
- $\mathrm{Dig}_h(X)$ = Gauss digitization of X with step h.

1.2 Concept of Meaningful Scales (2)

[redacted]

1. Exploit asymptotic properties of perfect shape digitizations.
   ⇒ Length (L) of maximal straight segments
2. They grow longer as h gets finer.
3. Estimate them from a multiresolution decomposition of the input shape.





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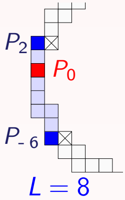

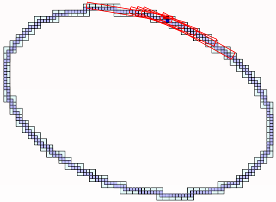

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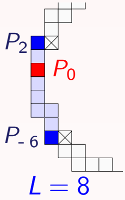

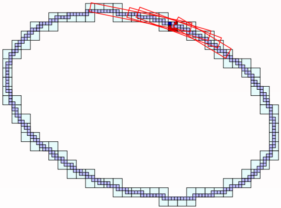

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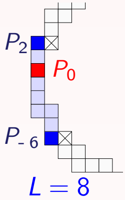

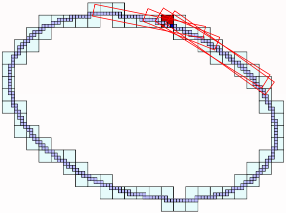

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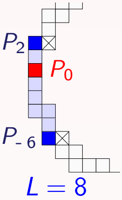

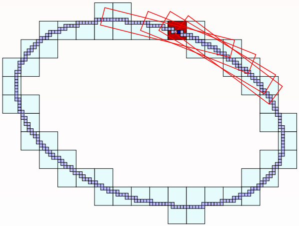

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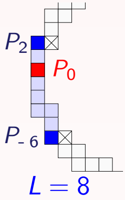

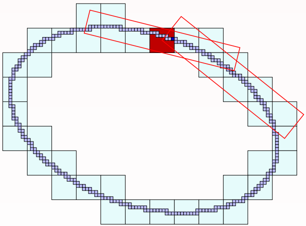

1.2 Concept of Meaningful Scales (3)

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- X simply connected shape in R^2 with piecewise C^3 boundary ∂X,
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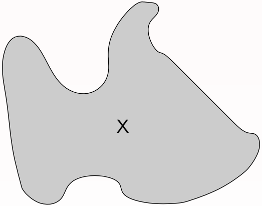

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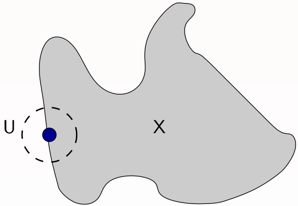

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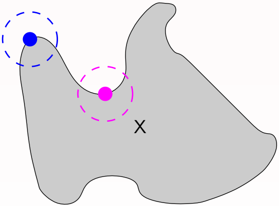
$$\partial X \cap U \text{ has null curvature, then } \Omega(1/h) \leq \quad L_j^h \quad \leq O(1/h) \tag{2}$$



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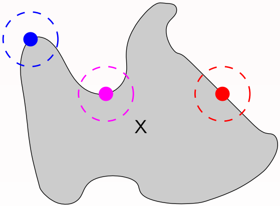

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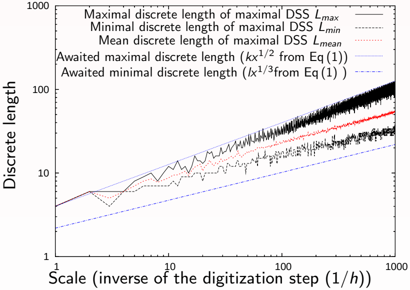

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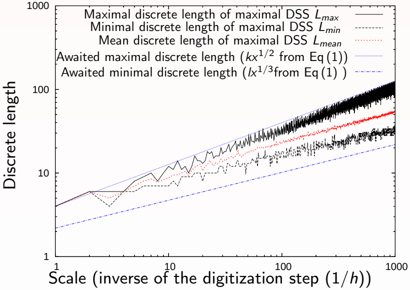
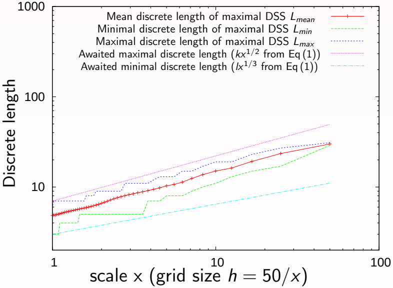


Experiments from subsampling



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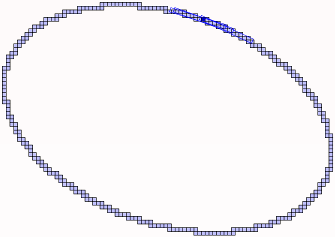
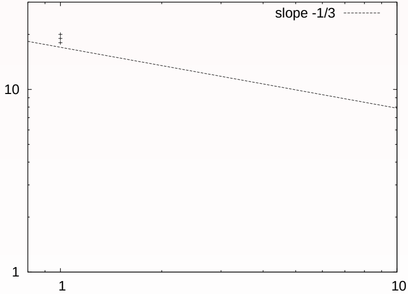
- Construction of a multiscale profile starting from initial resolution.
- Compare the multiscale profile to determine a local meaningful scale.
- Detect locally the amount of noise.
- Detect flat/curved parts.



$h_1 = h,\ L^{h_1} = (18, 20, 19)$



$(h, L(h))$ in log-scale

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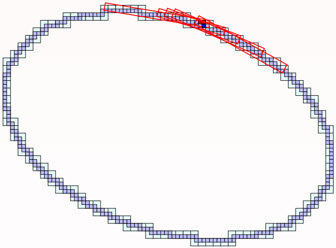
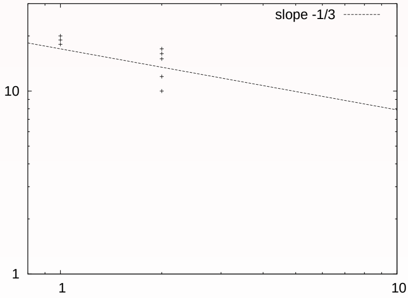


$h_2 = 2h,\ L^{h_2} = (15, 10, 12, 16, 17, \ldots)$



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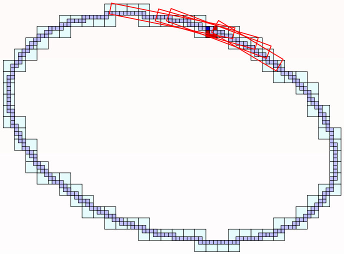
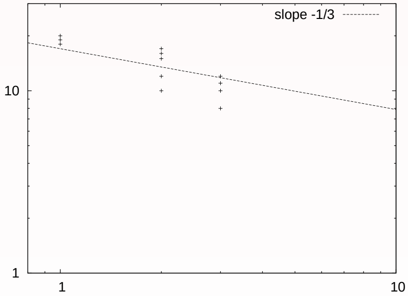


$h_3 = 3h,\ L^{h_3} = (12, 10, 11, 8, \ldots)$



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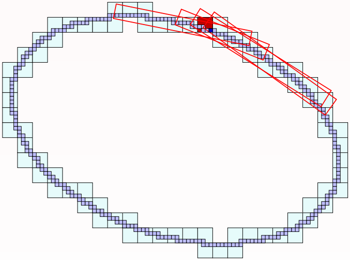
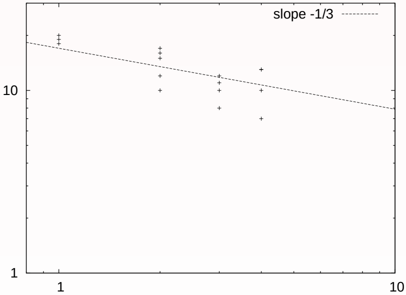


$h_4 = 4h,\ L^{h_4} = (10, 7, 13, 13, \ldots)$



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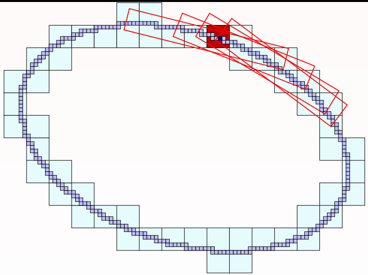
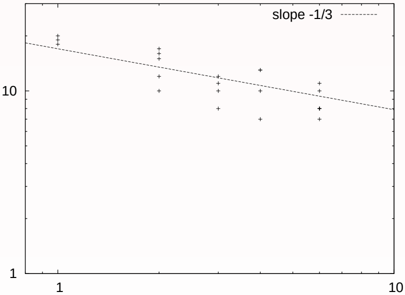


$h_6 = 6h,\ L^{h_6} = (8, 7, 8, 8, \dots)$



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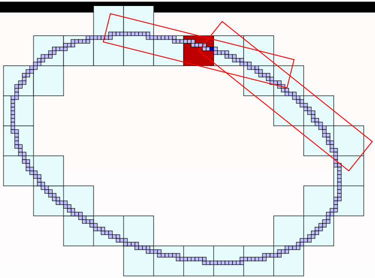
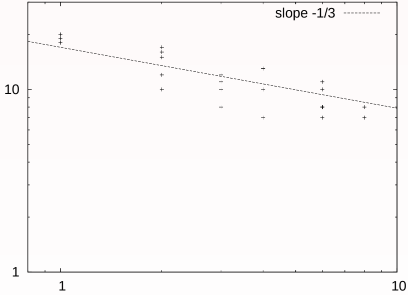


$h_8 = 8h,\ L^{h_8} = (8, 7, \ldots)$



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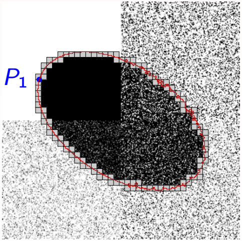

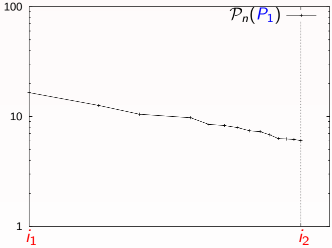

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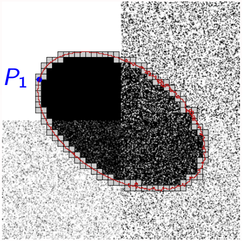

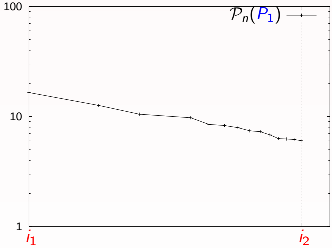

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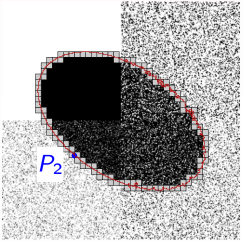

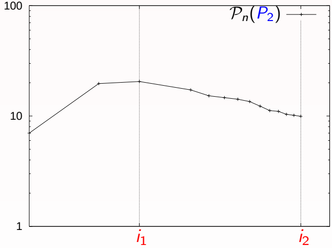

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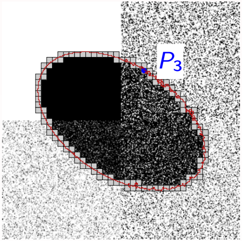

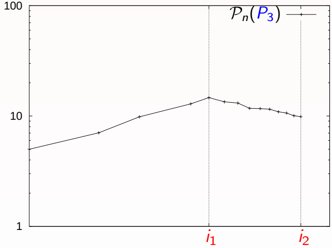

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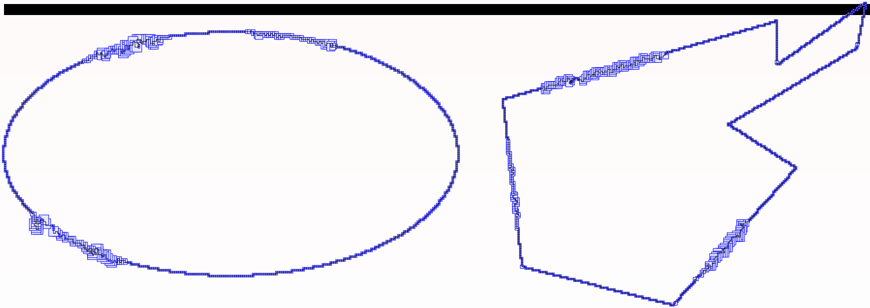

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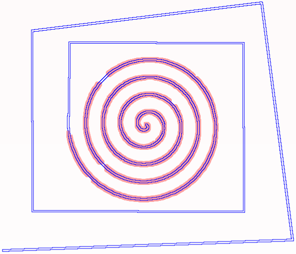

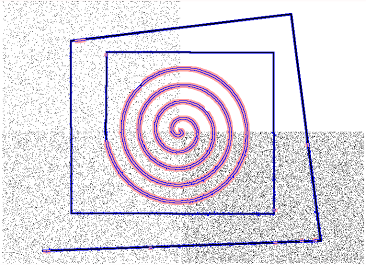

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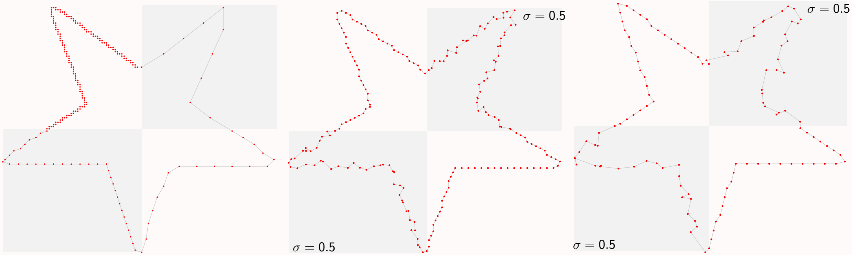

2. Meaningful Thickness

[redacted]

- Use another primitive to process non discrete set of points.
- α-Thick Blurred Segments [Faure et al., 2009, Debled-Rennesson et al., 2006]:
  - Defined with a thick parameter: t
  - maximal isothetic thickness of the convex hull.
    $\Rightarrow (P_1, Q_1, Q_2, P_2, P_3)$
- **Maximal** α-Thick Blurred Segments.
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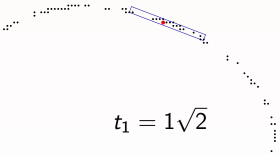

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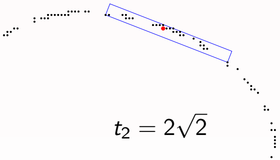

$t_2 = 2\sqrt{2}$

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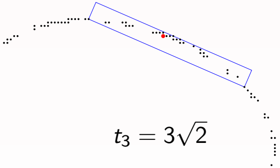

$t_3 = 3\sqrt{2}$

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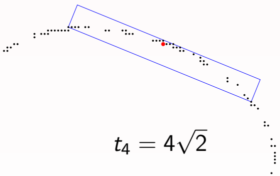

$$t_4 = 4\sqrt{2}$$

2. Meaningful Thickness

[redacted]

- Use another primitive to process non discrete set of points.
- α-Thick Blurred Segments [Faure et al., 2009, Debled-Rennesson et al., 2006]:
  - Defined with a thick parameter: t
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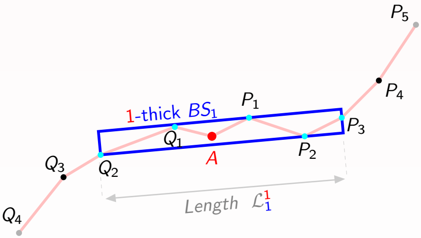

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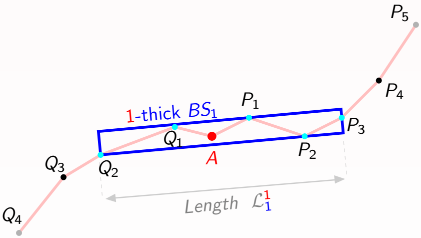

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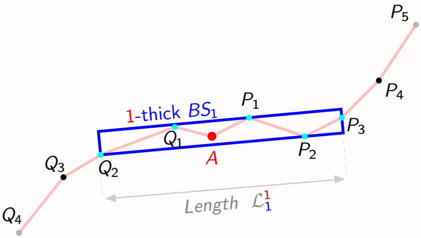
  $\Rightarrow$ the t step k given from the mean distance between each consecutive contour point.



Thickness asymptotic behaviour

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The plots of the lengths $\mathcal{L}_j^{t_i}/t_i$ in log-scale are approximately affine with negative slopes as specified besides:

	expected slope	
plot	curved part	flat part
$(\log(t_i),\log(\max_j \mathcal{L}_j^{t_i}/t_i))$	$\approx -\frac{1}{2}$	≈ -1
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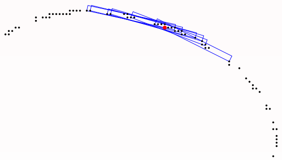

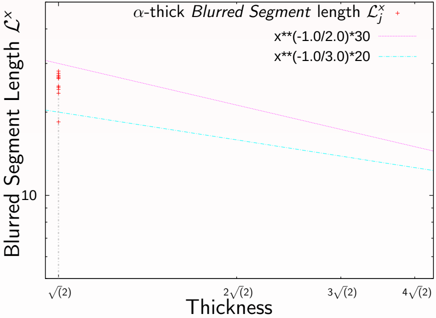

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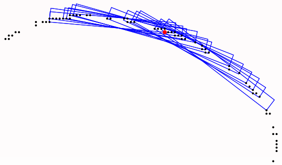

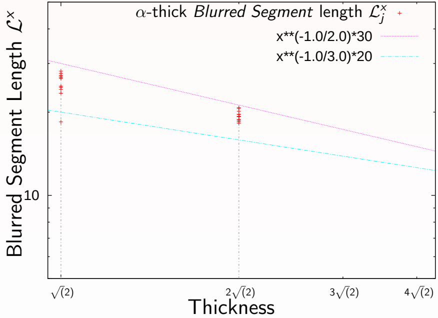

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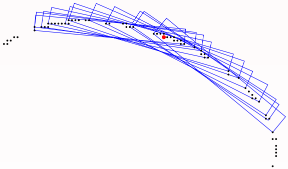

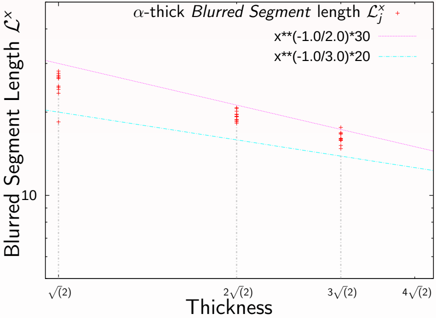

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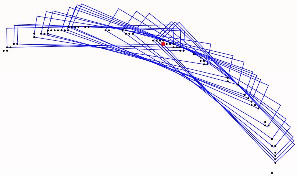

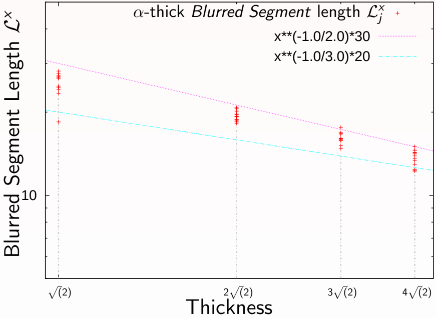

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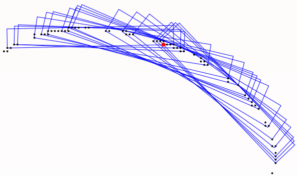

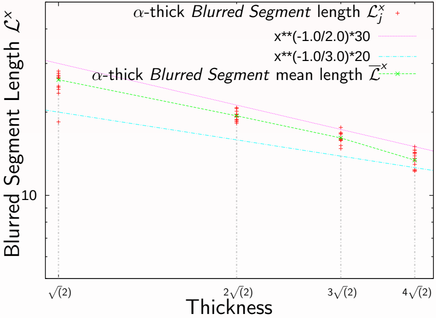

Multi-thickness criterion

[redacted]

The ***multi-thickness profile*** $\mathcal{P}_n(P)$ of a point P is defined as the graph $(\log(t_i), \log(\overline{\mathcal{L}}^{t_i}/t_i))_{i=1,\ldots,n}$.

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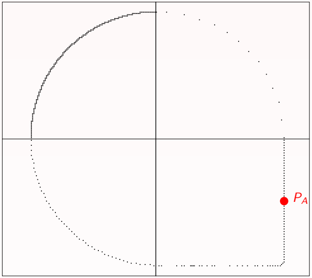
Defined for a point P on the boundary of a digital object as the slope coefficient of the simple linear regression of $\mathcal{P}_n(P)$.

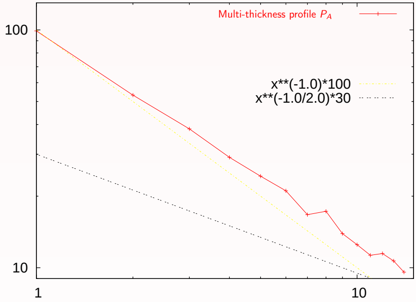
$\Rightarrow$ From previous Property:

- if P is in flat zone: $\mu_n(P)$ should be around -1.
- if P is in strictly convex or concave zone: $\mu_n(P)$ should be within $[-1/2, -1/3]$.

Illustration of multi-thickness profile

Example obtained form a shape with different sampling:





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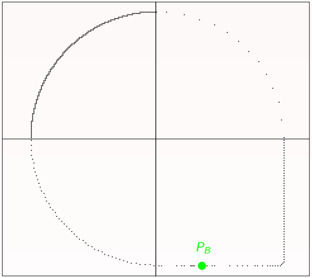

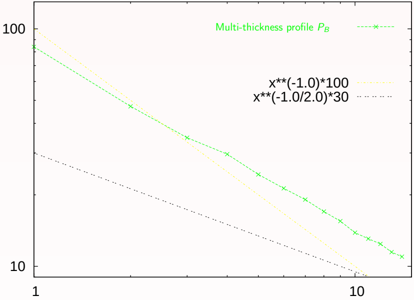

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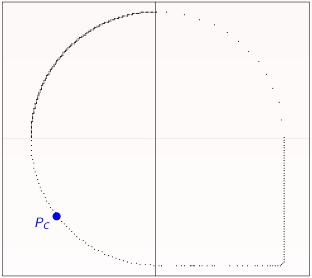

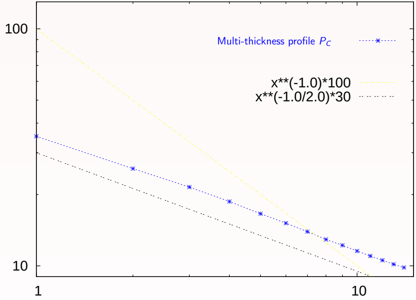

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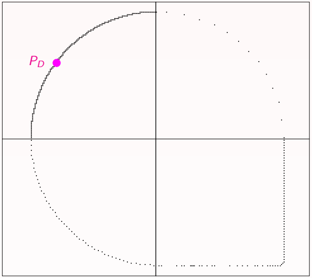

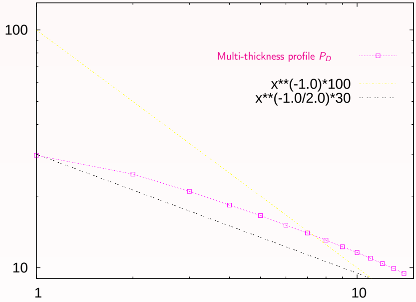

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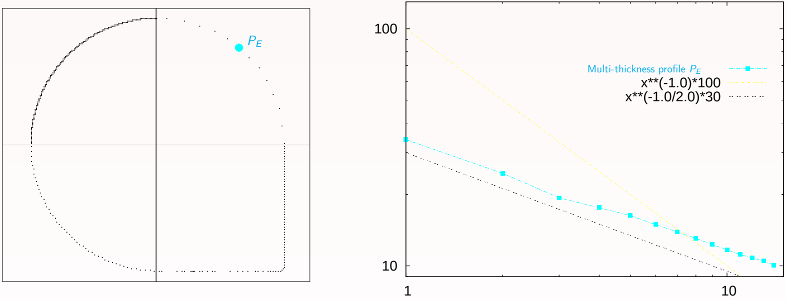

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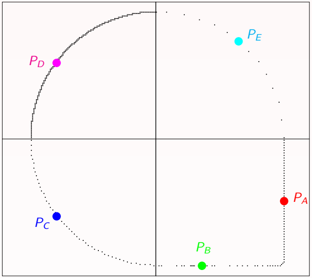


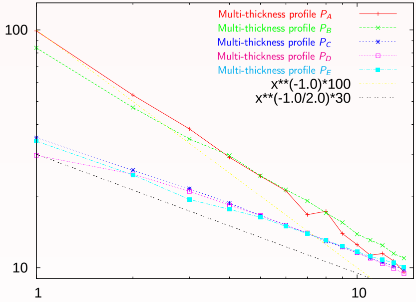
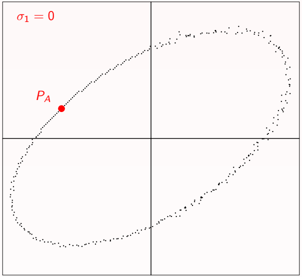
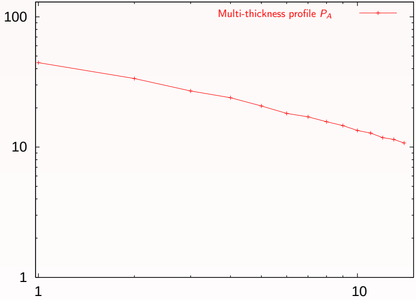


Illustration of multi-thickness profile (2)

Example obtained by adding noise:





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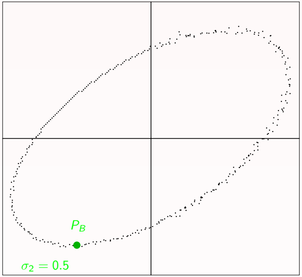

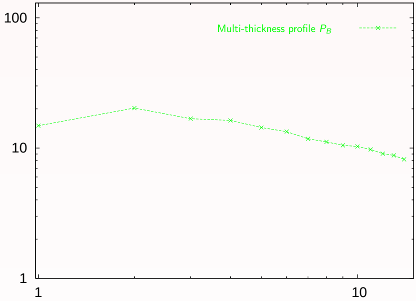

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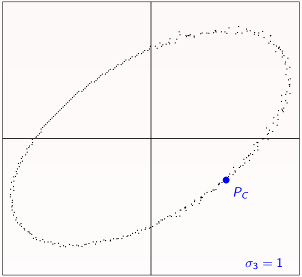

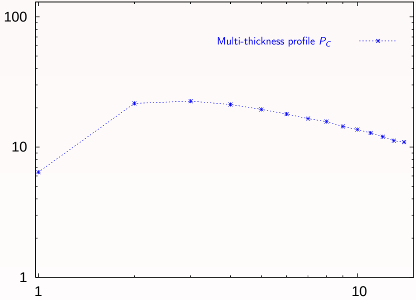

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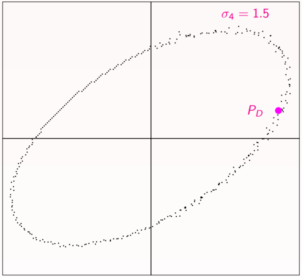

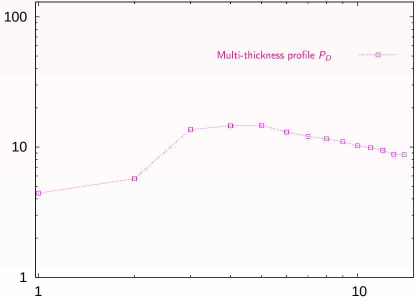

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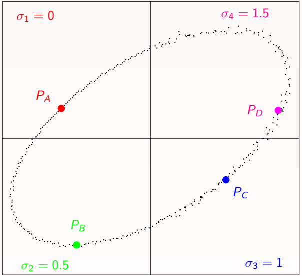

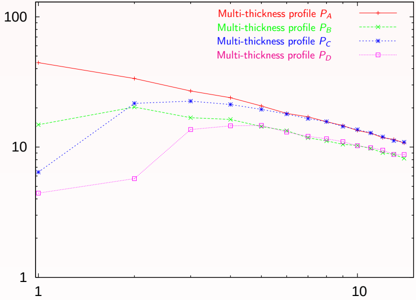

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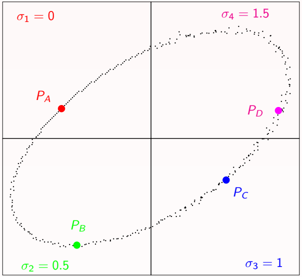

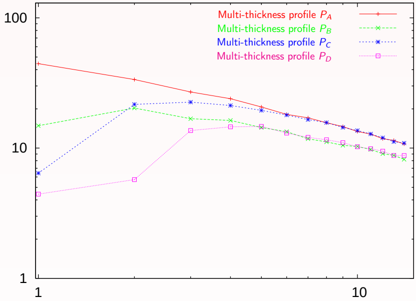

⇒ Define a noise threshold T_m to discriminate the curved and noisy zone.

Noise detection and Meaningful Thickness

[redacted]

A *Meaningful thickness* of a multi-thickness profile $(X_i, Y_i)_{1\leq i\leq n}$ is then a pair (i_1, i_2), $1 \leq i_1 < i_2 \leq n$, such that for all i, $i_1 \leq i < i_2$,

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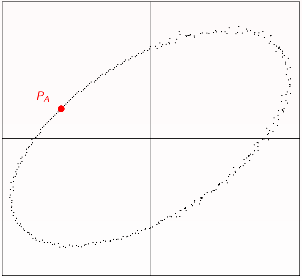
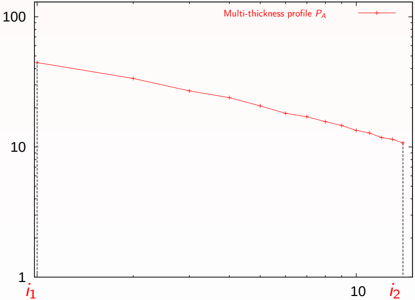
and the property is not true for $i_1 - 1$ and i_2.





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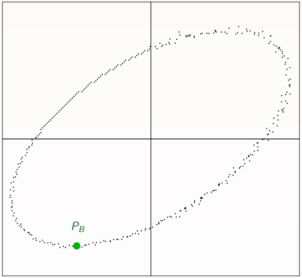

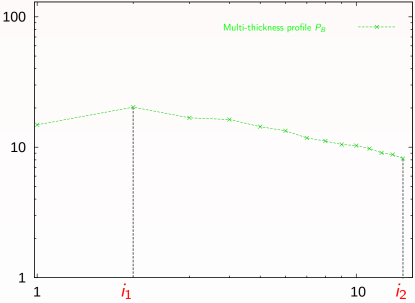

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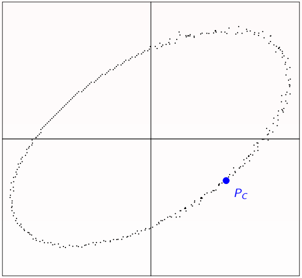

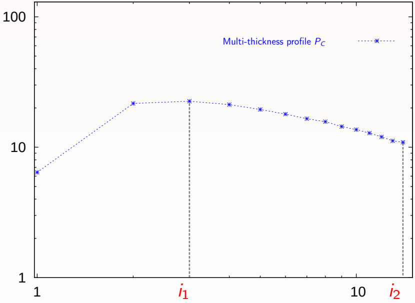

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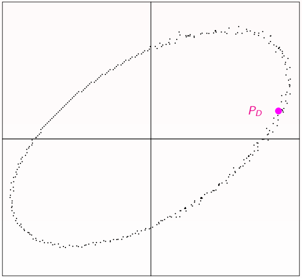

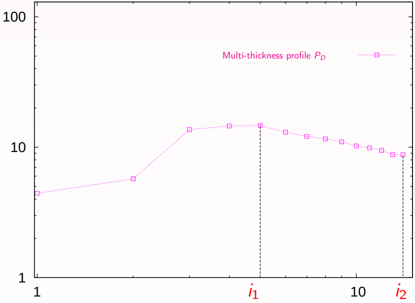

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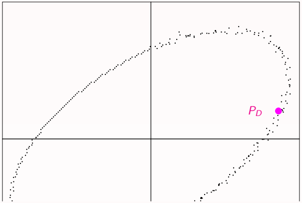
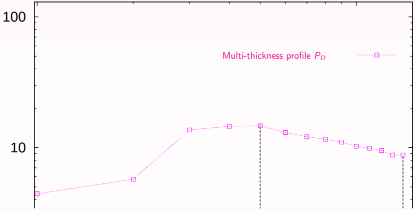
[redacted]



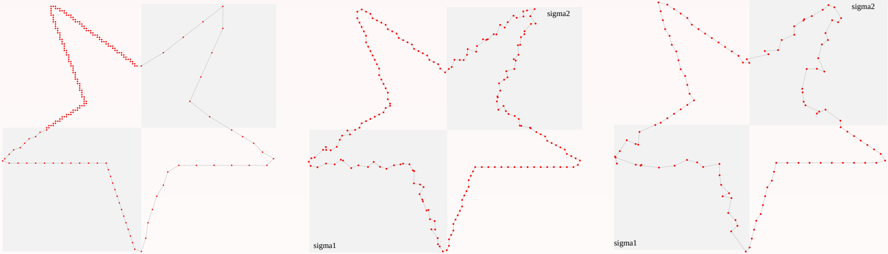


[redacted]

if (i_1, i_2) is the first meaningful scale at point P the noise level is $i_1 - 1$.

[redacted]

Experiments on polygonal shapes (1)



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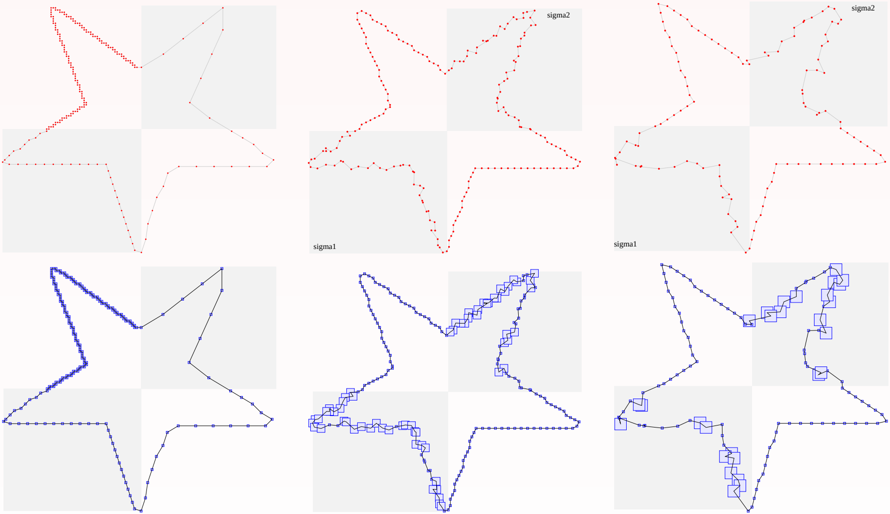
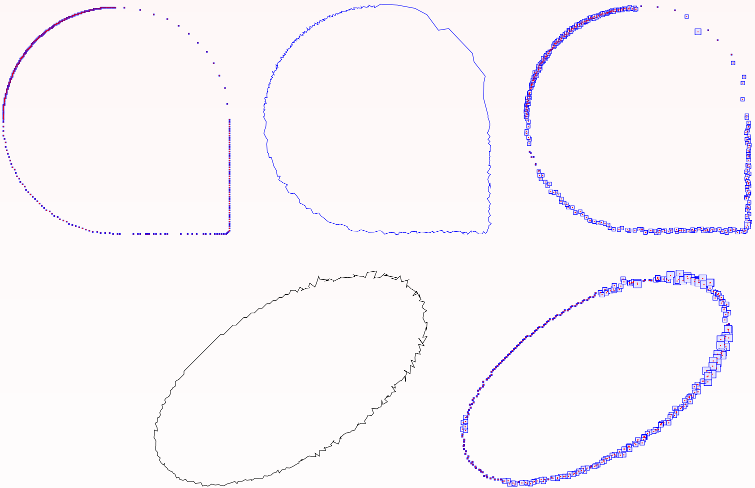
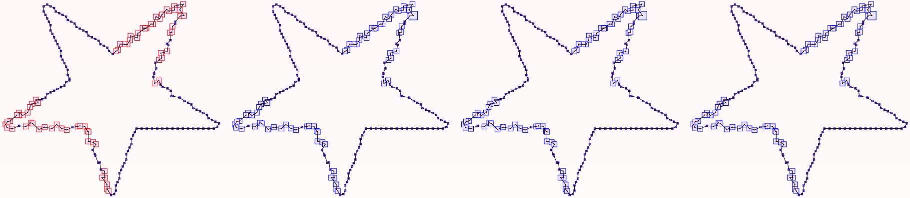


Experiments on polygonal shapes (2)



Stability from intern parameters

- Maximal thickness t_{max}



$t_{max} = 5\sqrt{2}$ $t_{max} = 10\sqrt{2}$ $t_{max} = 15\sqrt{2}$ $t_{max} = 20\sqrt{2}$

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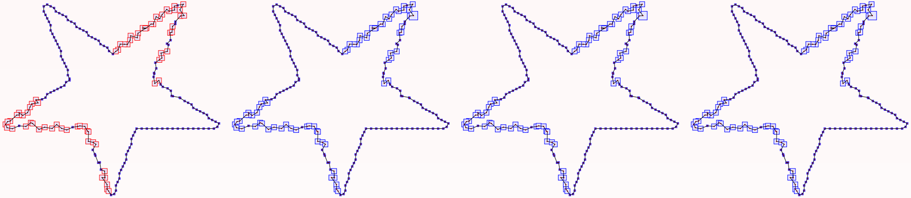
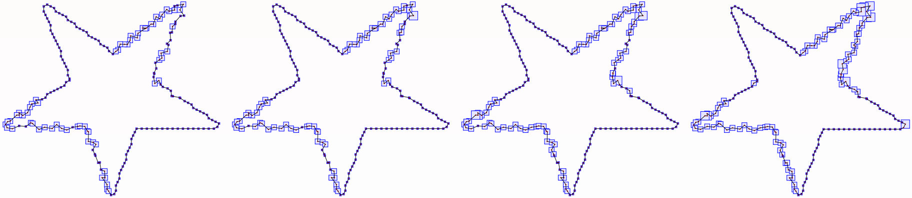
- Maximal thickness t_{max}



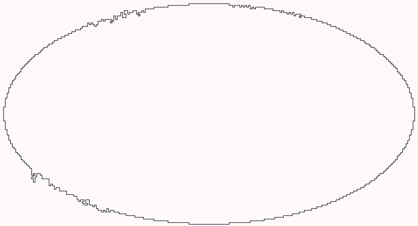
$t_{max} = 5\sqrt{2}$ $t_{max} = 10\sqrt{2}$ $t_{max} = 15\sqrt{2}$ $t_{max} = 20\sqrt{2}$

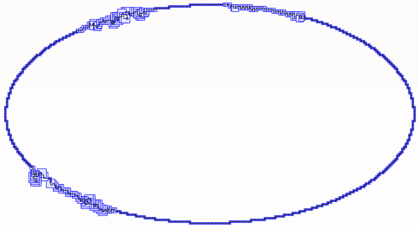
- Noise threshold T_m



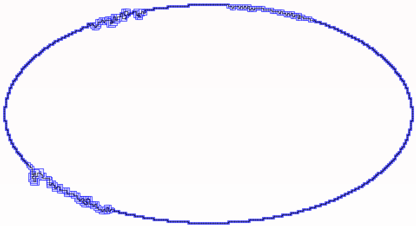
$T_m = 0.2$ $T_m = 0.0$ $T_m = -0.2$ $T_m = -0.4$

Comparison with the Meaningful Scales [Kerautret&Lachaud,09]



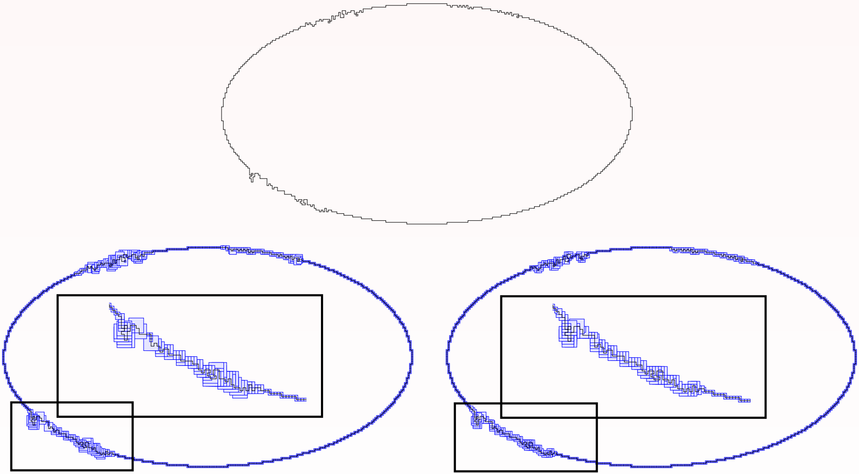


Meaningful scale



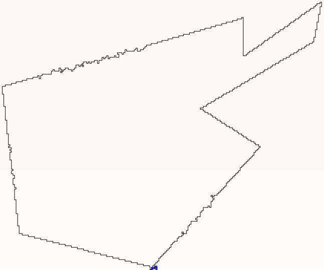
Meaningful thickness

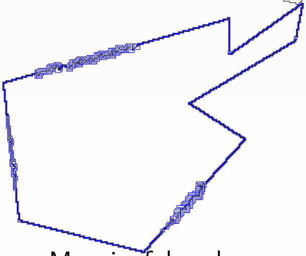
Comparison with the Meaningful Scales [Kerautret&Lachaud,09]



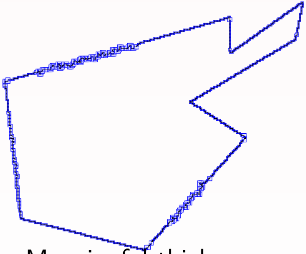
Meaningful scale | Meaningful thickness

Comparison with the Meaningful Scales (2) [Kerautret&Lachaud,09]



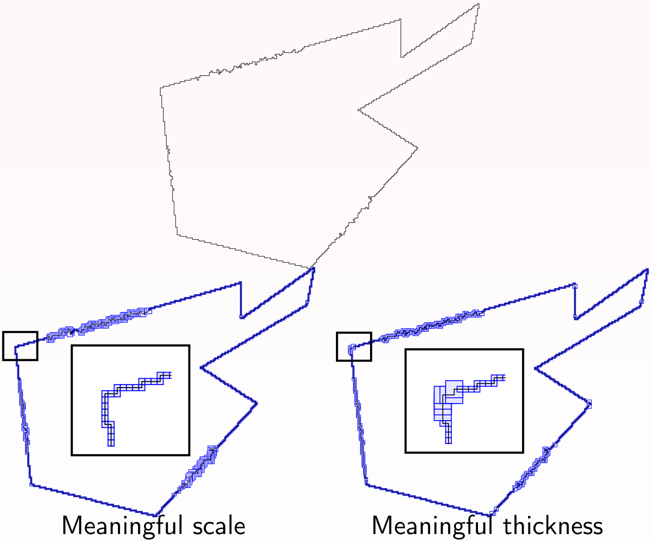


Meaningful scale

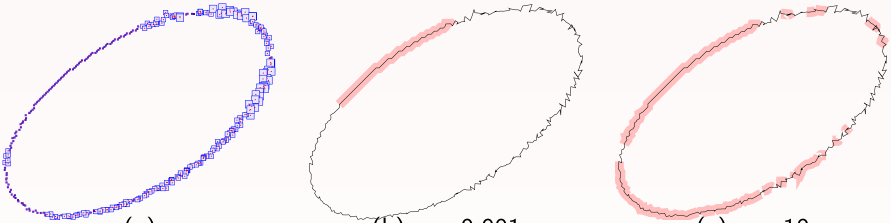


Meaningful thickness

Comparison with the Meaningful Scales (2) [Kerautret&Lachaud,09]



Comparison with the *Good Continuation* approach [Cao 03]



(a) (b) $\epsilon = 0.001$ [Cao 03] (c) $\epsilon = 10$ [Cao 03]

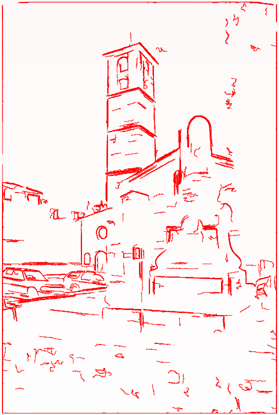
Simple applications (1)

[redacted]

- Extraction of all contours
- Apply meaningful thickness detection
- detection of straight parts.

source

contour from level set



meaningful parts

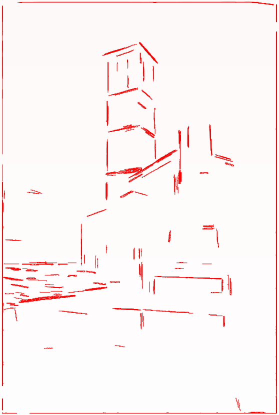
Simple applications (1)

[redacted]

- Extraction of all contours
- Apply meaningful thickness detection
- detection of straight parts.

source

contour from level set

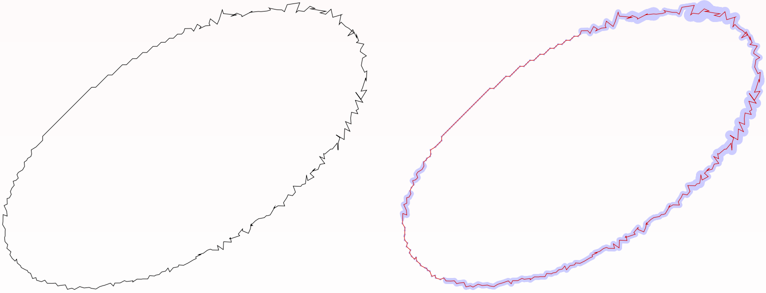


meaningful straight parts

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
- Constraints are also defined between polygon vertex by linear interpolation.



iteration 1

Simple applications (2)

[redacted]

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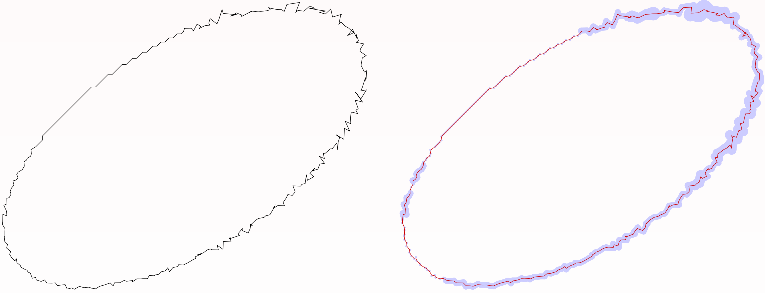

iteration 2

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
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  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
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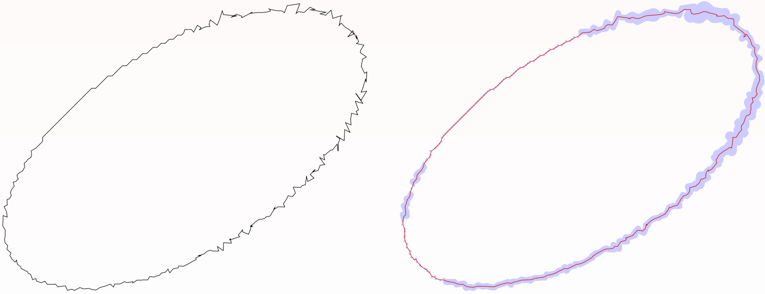

iteration 3

Simple applications (2)

[redacted]

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- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
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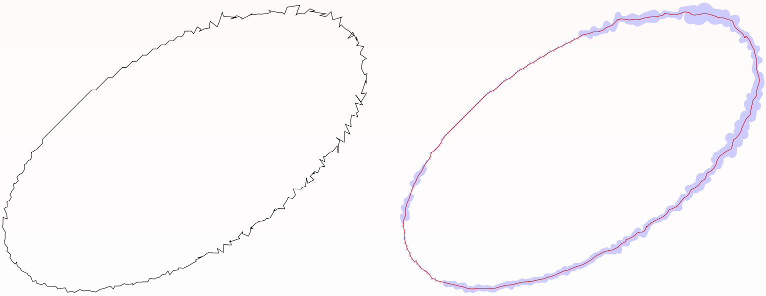

iteration 4

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
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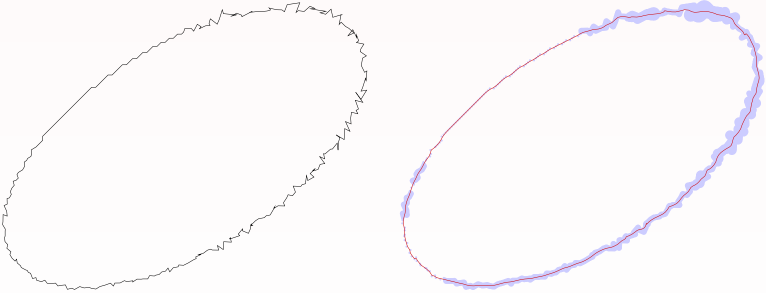
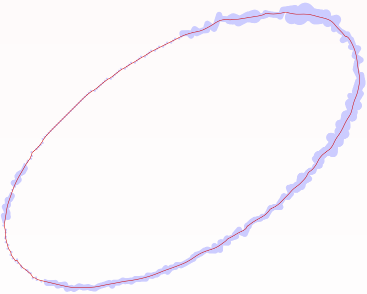


iteration 5

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
- Constraints are also defined between polygon vertex by linear interpolation.

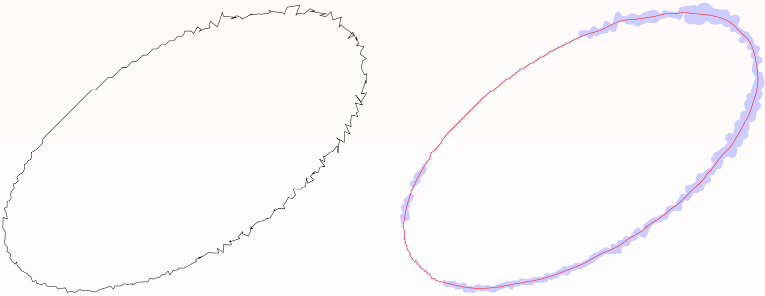


iteration 10

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
- Constraints are also defined between polygon vertex by linear interpolation.

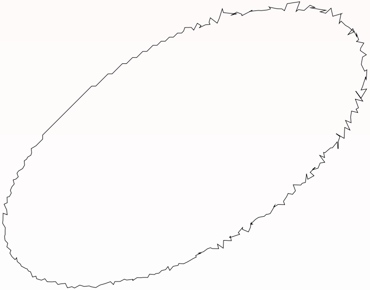


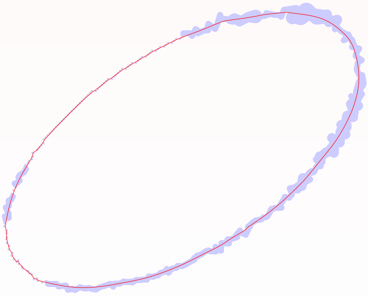
iteration 20

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
- Constraints are also defined between polygon vertex by linear interpolation.





iteration 50

Simple applications (2)

[redacted]

- Applying an iterative process on contour points P_i.
- Each points are moved:
  - by a weighted average of its two neighbors.
  - from constraint defined from meaningful thickness.
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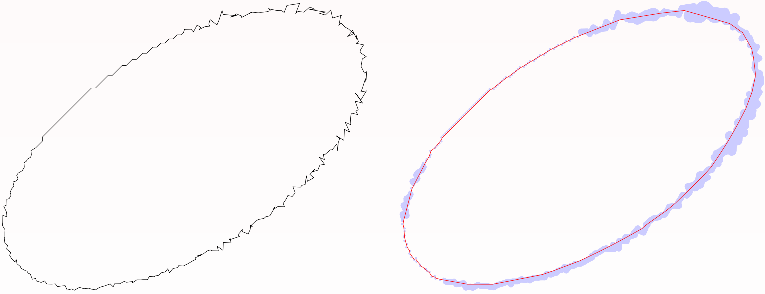
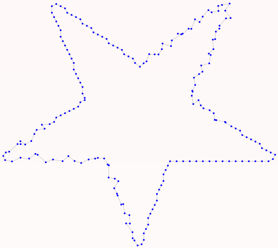
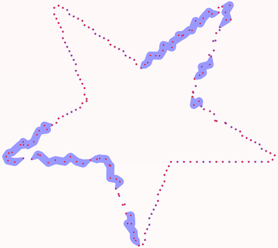


iteration 500

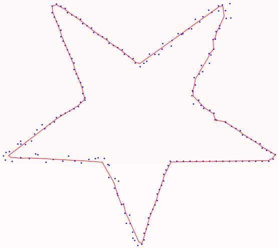
Simple applications (2)



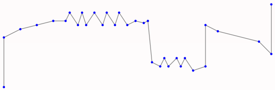
(d) source contour



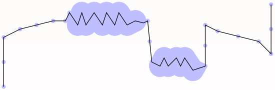
(e) meaningful thickness constraint



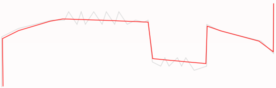
(f) resulting reconstruction



(g) source contour



(h) meaningful thickness constraint



(i) resulting reconstruction

Conclusion and discussion

[redacted]

- Simple to implement from the α-Thick Blurred Segments.
- Can be considered as parameter free.
- Equivalent quality for discrete data.
- Demonstration available online:
  http://kerrecherche.iutsd.uhp-nancy.fr/MeaningfulThickness

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[redacted]

- Theoretical bounds of asymptotic behavior.
- Meaningful scale detection on 3D discrete surfaces. ⇒ from the notion of 3D tangential cover [Charrier and Lachaud, 2011].
- Same principle on 3D surfaces mesh [Cuel, 2011].
- Add the Meaningful Thickness detection in .

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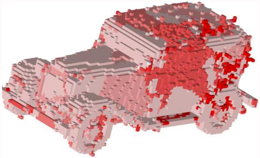

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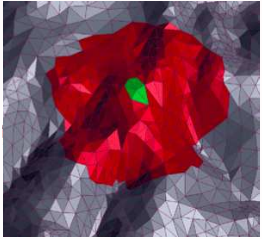

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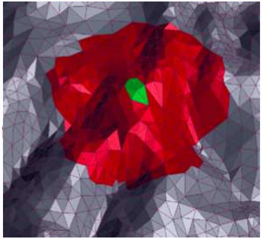

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