Circular arc reconstruction of digital contours with chosen Hausdorff error

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Abstract

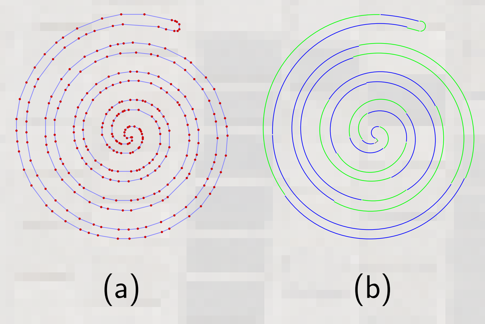
Instead of polygonalizing a contour, we propose to reconstruct the shape with circular arcs. To do so, we exploit the recent curvature estimators [1, 3]. From their curvature field, we introduce a new simple and efficient algorithm to approximate a digital shape with as few arcs as possible at a given scale, specified by a maximal admissible Hausdorff distance.

Keyword: Circular arc reconstruction, contour representation, curvature estimator.

1 Introduction

Objectives:

- Represent a contour with a efficient way than polygon.
- Less primitive with equivalent precision.
- To be able to include a scale parameter.



(a) $\delta_H = 2.018$ 288 segments

(b) $\delta_H = 2.011$ 27 arcs, 1 segment

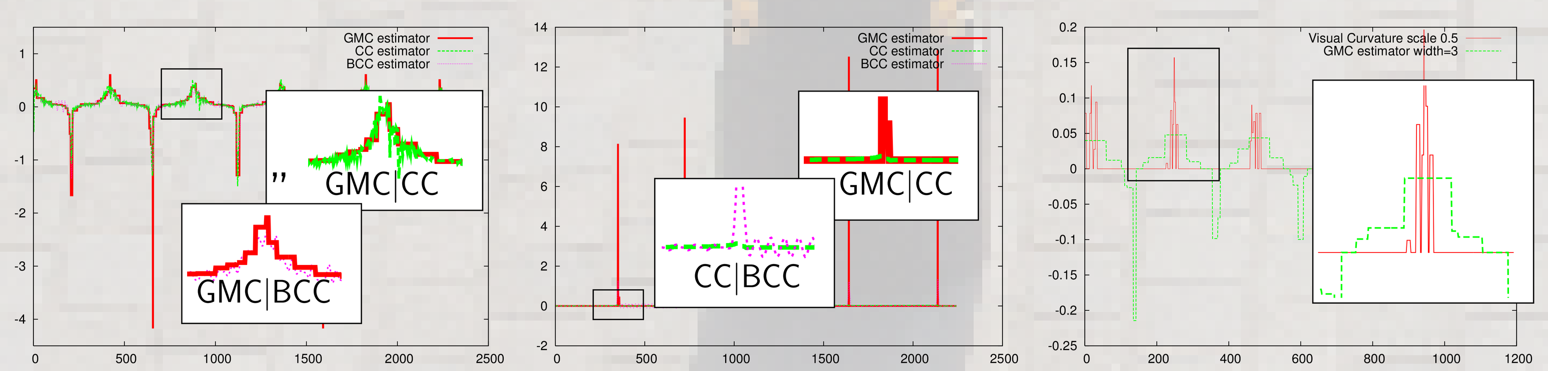
Main idea:

- Exploiting stable and robust to noise curvature estimators [1].
- Avoiding to use a curvature post processing to reduce parameters [4].
- Using only one parameter associated to the scale.

2 Curvature estimators

- **Global Minimisation Curvature (GMC).** [1]
  - Taking into account all the real shapes having the same digitization.
  - Minimise $\int \kappa^2 ds$.
- **Binomial Convolution Curvature (BCC).** [3]
  - Successive convolutions of m (say) binomial kernels and n
  - Alternative to the Gaussian smoothing technique (Scale controlled by m).
- **Visual Curvature (VC).** [2]
  - Measure the number of extreme points from a height function.
  - Filter non-significant features at a given scale (Only qualitative estimation).

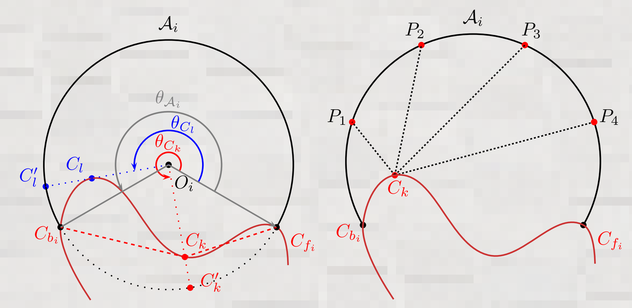
Illustration and comparison of curvature estimators:



3 Contour Reconstruction with Circle Arcs

Main steps of the algorithm:

- Decompose curvature estimation to extract significant curved areas.
- Split/merge strategy (given maximal error E_{max})

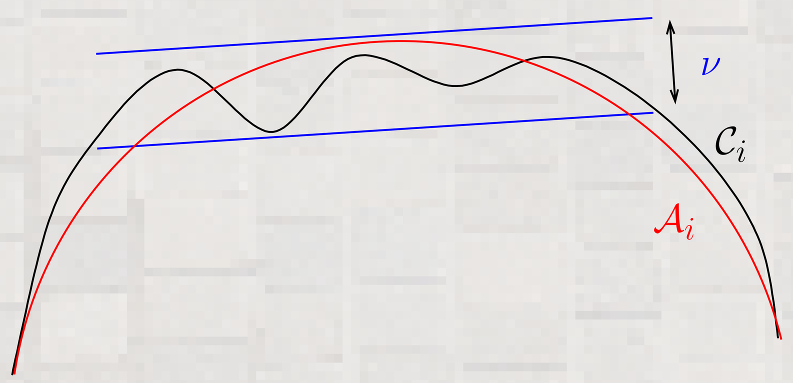


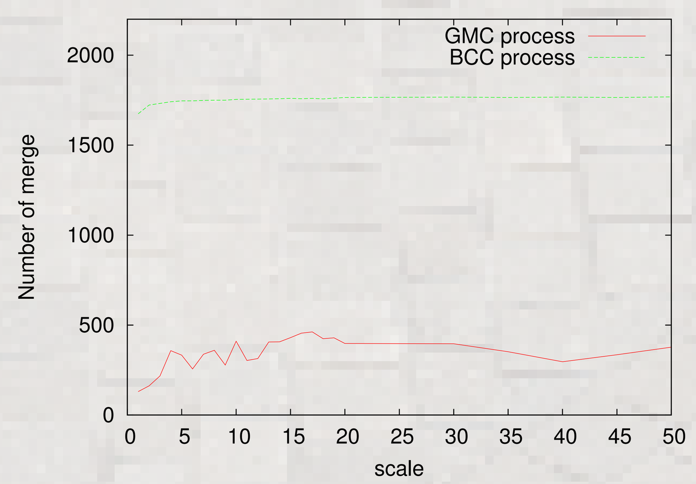
Use an approximation of the hausdorff:

$$\delta_H(\mathcal{A}_i,\mathcal{C}_i) = \max\{\max_{b\in\mathcal{C}_i}\{\min_{a\in\mathcal{A}_i} d(a,b)\}, \max_{a\in\mathcal{A}_i}\{\min_{b\in\mathcal{C}_i} d(a,b)\}\}$$

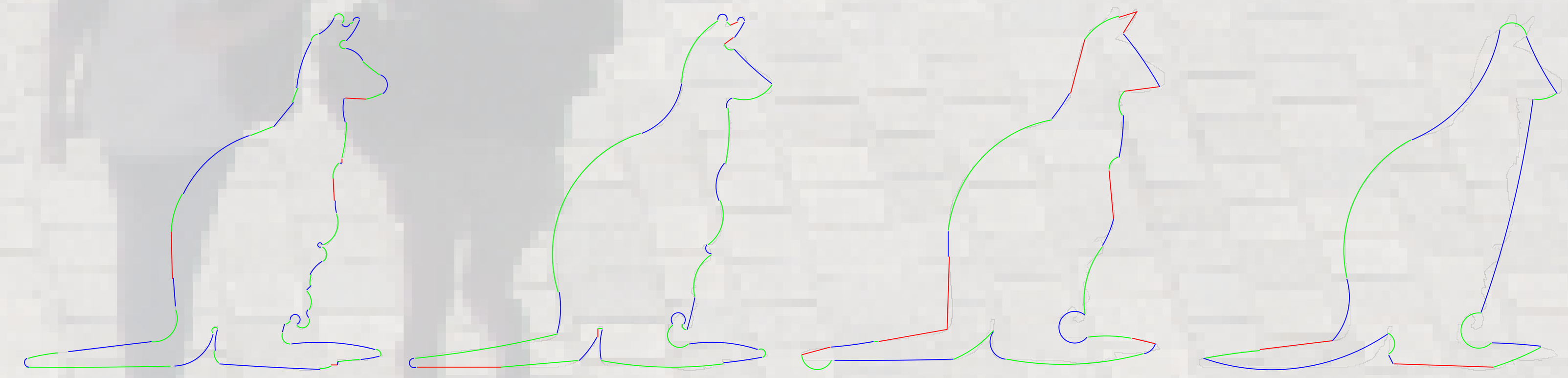
Main algorithm

Data: $C=\{C_i\}_{i=0}^n$ digital curve, $\kappa=\{\kappa_i\}_{i=0}^n$ curvature estimation, float maxArcError;
Result: curve represented by a set of arcs and segments.
begin
Decompose κ into a set of constant curvature interval S defined by: $\{(b_0,f_0),...,(b_i,f_i),...,(b_M,f_M)\}$.
For each contour point C_i, store in $regionIndex[i]$ the index $k\in\{0,..M\}$ of its region $S[k]$.
Extract from S the set S_m containing all the regions which are a local maxima/minima.
$S_{tmp}=S$;
while $nbElements(S_{tmp})!=0$ **do**
  S_{tmp} = SPLIT_REGIONS(S_{tmp}, S, $regionIndex$, maxArcError);
// First extension from mini/maxima regions:
while $nbElements(S_m)!=0$ **do**
  S_m = EXTEND_PLOT_REGIONS(S_m, S, regionIndex, κ, maxArcError);
// Second extension from all others regions S_u:
S_u= set of index of valid non maxima/minima regions of the current regions.
while $nbElements(S_u)!=0$ **do**
  S_u = EXTEND_PLOT_REGIONS(S_u, S, regionIndex, κ, maxArcError);
// Verify or change primitive for region which are better approximate
// by a straight segment:
checkBestPrimitive(S, tabRegionIndex, maxArcError);
end

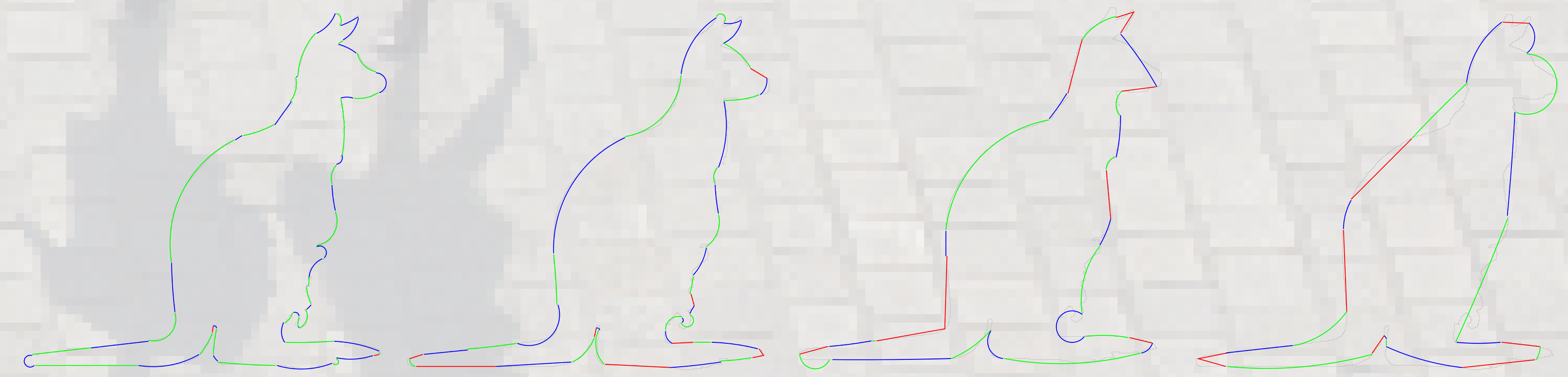




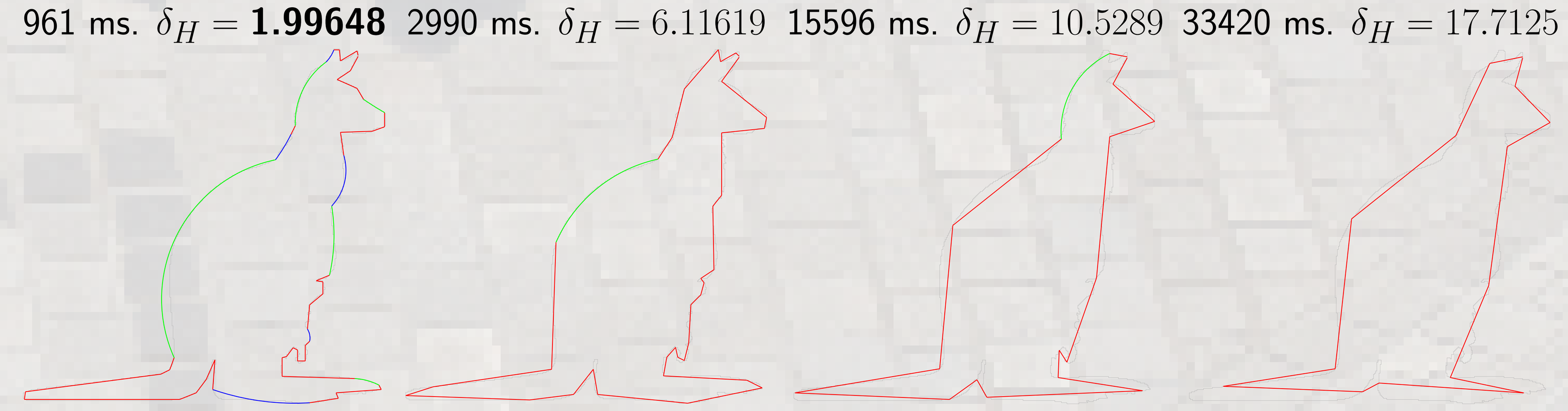
4 Experiments



GMC: $\overline{A}=53$, $\overline{S}=5$ 184 ms. $\delta_H=2.06539$ | GMC: $\overline{A}=31$, $\overline{S}=4$ 261 ms. $\delta_H=\mathbf{3.94931}$ | GMC: $\overline{A}=17$, $\overline{S}=3$ 464 ms. $\delta_H=\mathbf{10.2849}$ | GMC: $\overline{A}=14$, $\overline{S}=2$ 619 ms. $\delta_H=\mathbf{17.2402}$



BCC $\overline{A}=53$, $\overline{S}=2$ 961 ms. $\delta_H=\mathbf{1.99648}$ | BCC $\overline{A}=33$, $\overline{S}=9$ 2990 ms. $\delta_H=6.11619$ | BCC $\overline{A}=20$, $\overline{S}=9$ 15596 ms. $\delta_H=10.5289$ | BCC $\overline{A}=15$, $\overline{S}=8$ 33420 ms. $\delta_H=17.7125$

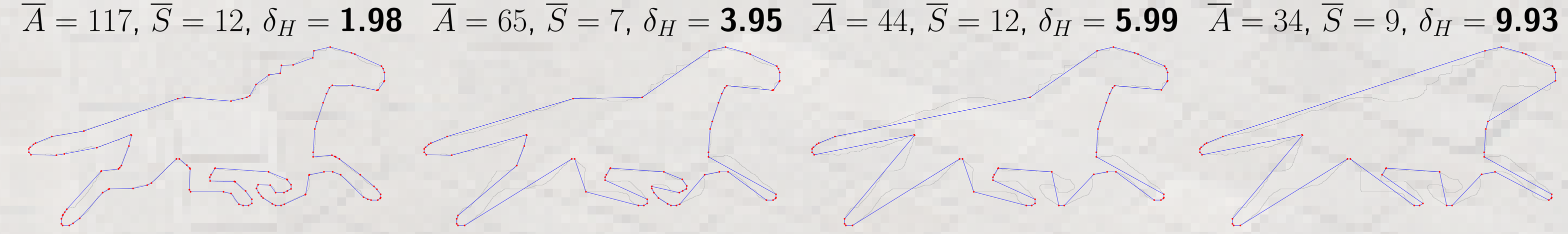


NASR $\overline{A}=10$, $\overline{S}=40$ 115 ms. $\delta_H=8.81655$ | NASR $\overline{A}=1$, $\overline{S}=34$ 138 ms. $\delta_H=9.84886$ | NASR $\overline{A}=1$, $\overline{S}=16$ 171 ms. $\delta_H=19.6977$ | NASR $\overline{A}=0$, $\overline{S}=14$ 190 ms. $\delta_H=32.8938$

GMC

$\overline{A}=117$, $\overline{S}=12$, $\delta_H=\mathbf{1.98}$ | $\overline{A}=65$, $\overline{S}=7$, $\delta_H=\mathbf{3.95}$ | $\overline{A}=44$, $\overline{S}=12$, $\delta_H=\mathbf{5.99}$ | $\overline{A}=34$, $\overline{S}=9$, $\delta_H=\mathbf{9.93}$

VC



$\overline{S}=128$, $\delta_H=11.58$ | $\overline{S}=72$,$\delta_H=26.22$ | $\overline{S}=55$,$\delta_H=35.44$ | $\overline{S}=33$, $\delta_H=81.62$

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