Robust Estimation of Curvature along Digital Contours with Global Optimization

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Abstract
We present a new curvature estimator based on global optimisation. This method called **G**lobal **M**in-**C**urvature exploits the geometric properties of digital contours by using local bounds on tangent directions defined by the maximal digital straight segments. The estimator is adapted to noisy contours by replacing maximal segments with maximal blurred digital straight segments.

Keyword: curvature estimator, blurred segments, noise.

1 Introduction

If G is a geometric feature defined for a family $\mathbb{F}$ of shape in $\mathbb{R}^2$, the estimator E_G is multigrid convergent towards G iff for any shape $X \in \mathbb{F}$, there exists some $h_X > 0$ for which:

$$\forall h, 0 < h < h_X, \|E_G(\mathrm{Dig}_h(X)) - G(X)\| \leq \tau(h),$$

where $\tau : \mathbb{R}^+ \to \mathbb{R}^{+*}$ has a limit value 0 at $h = 0$ and it defines the speed of convergence of E_G toward G.

Several critics:
- Precision only guaranteed for high resolution.
- Convergence obtained for perfect digitization process.

Vectorial generation		
200 dpi	300 dpi	400 dpi
nt	nt	nt
Result of scan		
200 dpi	300 dpi	400 dpi
nt	nt	nt

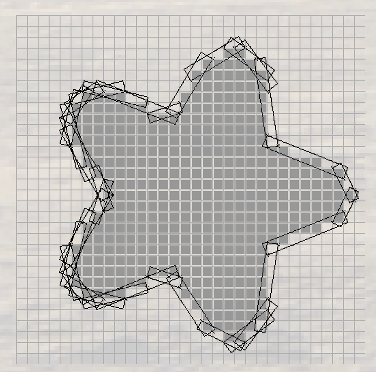
Objective:
- Obtain a good precision even with coarse resolution.
- Adapted to shape not perfectly digitized or noisy.

Main idea:
- Taking into account all the real shapes having the same digitization.
- Restricting the estimation which corresponds to more probable shape.
- Best length estimator: minimise $\int ds$ [5]

Curvature estimator: minimise $\int \kappa^2 ds$

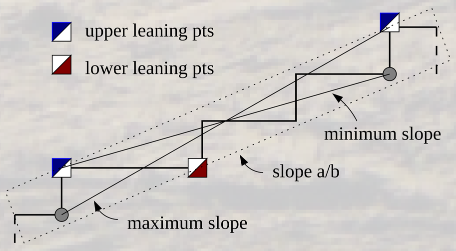
2 Tangential Cover and Tangent Space

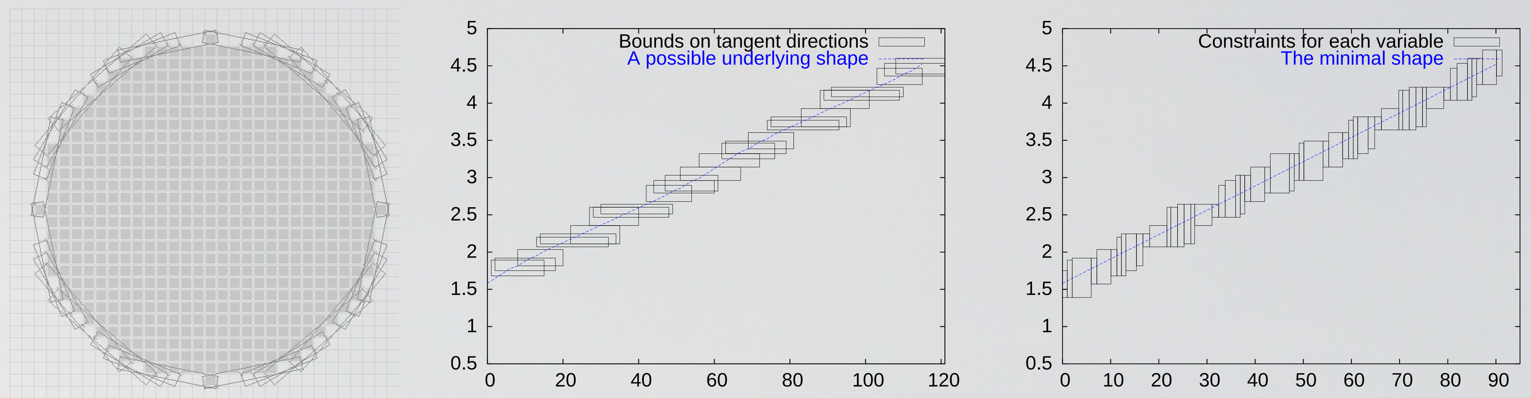
Definition: maximal segment
By denoting $S(i,j)$ the predicate "$C_{i,j}$ is a digital straight segment", a *maximal segment* of C is a sequence $C_{i,j}$ such that: $S(i,j) \wedge \neg S(i,j+1) \wedge \neg S(i-1,j)$.
The tangential cover is defined by the set of maximal segments contained in a digital contour.



Bounds on the tangent direction
- Minimal and maximal directions defined from the upper and lower leaning points.
- For a maximal segment of characteristics (a,b) we have:

$$p_{min} = \frac{a}{b} - \frac{1}{b} \quad \text{and} \quad p_{max} = \frac{a}{b} + \frac{1}{b}$$





3 Curvature Estimation by Optimisation

Principle:
- Shape of reference extracted from tangent space representation.
- Its geometry is entirely defined by the mapping $\theta_{\mathcal{C}}$ which associates to an arc length s the direction of the tangent at point $\mathcal{C}(s)$ ($\theta_{\mathcal{C}} = \angle(0x, \vec{\mathcal{C}'})$).
- Since the curvature is the derivative of the tangent direction, the integral $J[\mathcal{C}]$ along $\mathcal{C}$ of its squared curvature is then

$$J[\mathcal{C}] = \int_{\mathcal{C}} \kappa^2 = \int_0^L \kappa^2(s)ds = \int_0^L \left(\frac{d\theta_{\mathcal{C}}}{ds}\right)^2 ds. \quad (1)$$

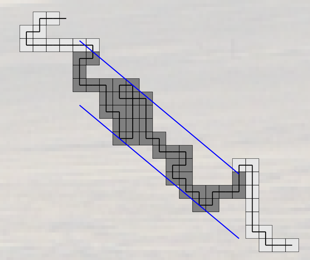
Curvature estimation of reference shape to O is thus reduced to

$$\text{Find } (t_l)_l, \text{ which minimizes } J[\mathcal{C}[\ldots,t_l,\ldots]] = \sum_l \left(\frac{t_{l+1}-t_l}{s_{i_{l+1}} - s_{i_l}}\right)^2 (s_{i_{l+1}} - s_{i_l}),$$
$$\text{subject to} \quad \forall l, a_l \leq t_l \leq b_l.$$

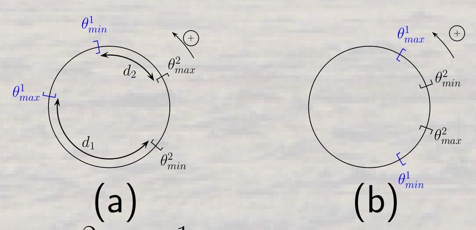
We use classical iterative numerical techniques to solve this optimization problem, simply following $\frac{\partial J}{\partial t_l}$ for each variable t_l while staying in the given interval. Geometrically, each variable t_l is moved toward the straight segment joining $(s_{i_{l-1}}, t_{l-1})$ to $(s_{i_{l+1}}, t_{l+1})$. The *GMC estimator* E_κ^{GMC} is then simply defined as the derivative of the piecewise linear function joining points (s_{i_l}, t_l), rescaled by h.

4 Adaptation to Blurred Segments

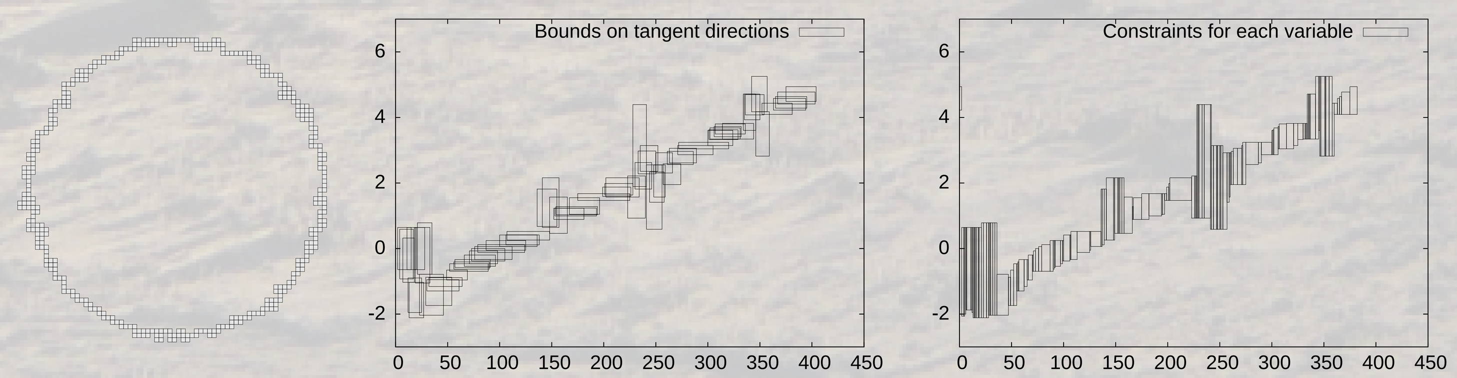
Maximal blurred segments
- Recognition algorithm proposed by Debled *et al.* [1] .
- Removing of the restrictive hypothesis which assumes that points are added with increasing x coordinate (as Roussillon *et al.* [4]).



Bounds on tangent direction of blurred segment
- Taking $min(\theta^i_{min})$ and $max(\theta^i_{max})$:
  $\Rightarrow$ not always consistent with intervals of size superior to π.
- Merging process of the different intervals defined according several configurations: examples of fusion configurations: $I_1 = [\theta^1_{min}, \theta^1_{max}]$ and $I_2 = [\theta^2_{min}, \theta^2_{max}]$.

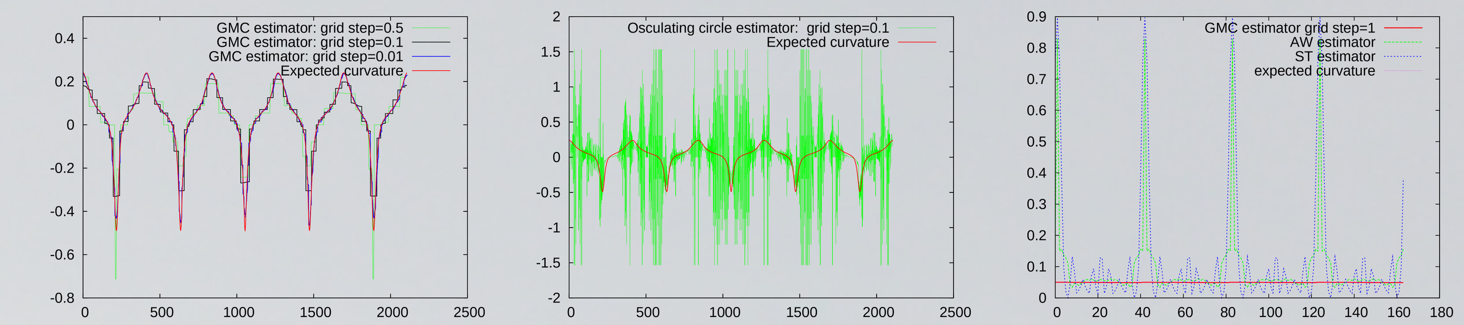


(a) $I = [\theta^2_{min}, \theta^1_{max}]$ (b) $I = [0, 2\pi]$



5 Results

Results and comparison on smooth shapes

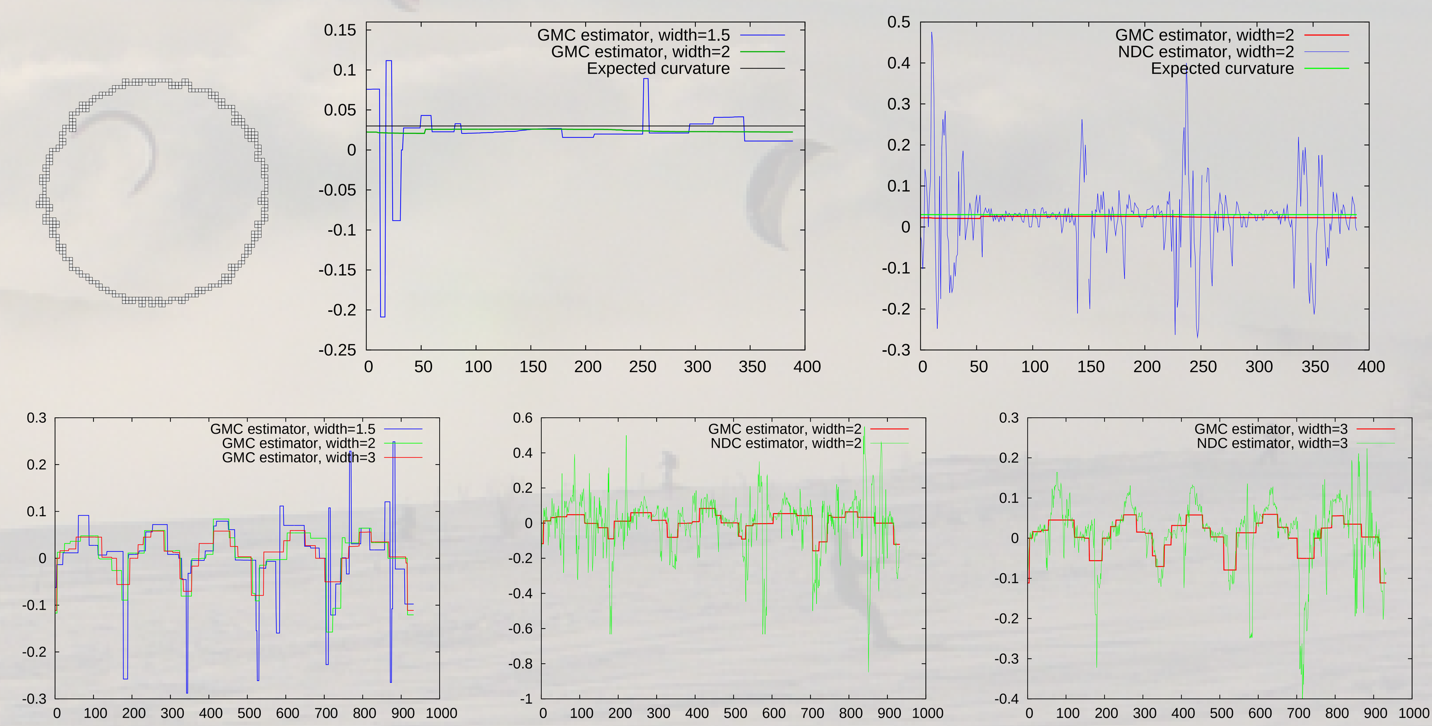


Comparison obtained from the flower of Sect. 2. Comparison with osculating circle estimator. Comparison with **Symetric Tangent** and **Adaptative Window** estimator.

Curvature extraction on noisy shapes
- Noisy shape generation (based on the document degradation model of Kanungo [2]):

Initial shape. Noise added according to the distance to the shape. Connected component. Extracted contour.

- Results on noisy circle and flower and comparison with **N**guyen and **D**ebled **C**urvature estimator [3]:



6 Conclusion

This new estimator shows precise results with different grid sizes. By replacing digital straight segments with blurred segments, the estimator is robust to noise and gives better results than other methods. Moreover it allows to detect easily inflection points and maximal curvature areas.

References

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