Experimental Comparison of Continuous and Discrete Tangent Estimators Along Digital Curves.

François de Vieilleville   Jacques-Olivier Lachaud

SUNY, USA
LAMA, Université de Savoie, France

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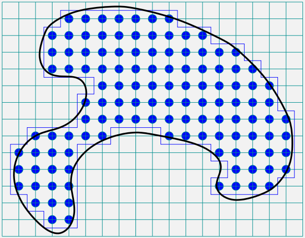
Outline

Digital Objects - Digital Curves

- Digital plane $\equiv \mathbb{Z}^2$.
- Digital object $\equiv$ Finite set of points of $\mathbb{Z}^2$.
- From euclidean shapes to digital shapes : $\mathcal{E} = \mathcal{X} \cap h\mathbb{Z} \times h\mathbb{Z}$.

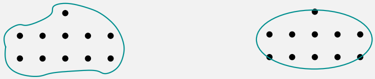


The digital boundary can be extracted using inter-pixel contours or cellular decomposition.

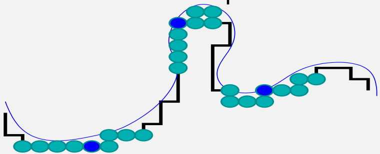
Elements of the digital curve are ordered and indexed.
$\rightarrow$ Computation of local geometric characteristics use windows containing finite number of points.

Problems When Analyzing Digital Curves

(1) There exists infinitely many continuous Euclidean shapes for a digitized shape.



(2) How to determine the size of computation window w.r.t. local geometry ?



(3) Time to spend on computations may be limited.

(4) The digitized curve can be noisy or damaged.

Solution 1 : Continuous Estimators and Fixed Sized Window

(1) Add hypotheses for the reference shape : smoothness, compactness, convexity, minimal perimeter or maximal area.

→ Brings bias in estimation : special underlying curve.

(2 & 3) Fix the size of the computation window.

→ *explicit* trade-off between time computation and precision, loss of local geometry consistency.

(4) Well defined, use smoothing or averaging.

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→ For tangent estimation we chose Gaussian derivative and low order polynomial fitting.

Solution 2 : Use Maximal Digital Straight Segment Based Algorithms

(1) No hypotheses on the reference shape : study of the digital linear part of the digital object.

(2 & 3) Size of the computation window computed automatically.

→ At very low resolution, consistency with local geometry ?

(4) No proper definition. Use special weighted averaging, Maximum Blurred Segments ?

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→ For tangent estimation we choose the λ-MST tangent estimator.

Polynomial Fitting

- $(C_i = (x_i, y_i))_{1 \leq i \leq 2q+1}$ set of $2q+1$ consecutive samples on digital curve.
- Minimize the functional : $E(a_0, \ldots, a_N) = \sum_{i=1}^{2q+1} \left(y_i - \sum_{j=0}^{j=N} a_j x_i^j \right)^2$.
- $$\begin{aligned} E_{LR}(a,b) &= E(a,b,0,\ldots) && \text{Linear Regression,} \\ E_{IPF}(a,b) &= E(0,a,b,0,\ldots) && \text{Implicit Parabola Fitting,} \\ E_{EPF}(a,b,c) &= E(a,b,c,0,\ldots) && \text{Explicit Parabola Fitting.} \end{aligned}$$
- Estimation on whole curve in $\mathcal{O}\left((2q+1) * \text{number of points}\right)$

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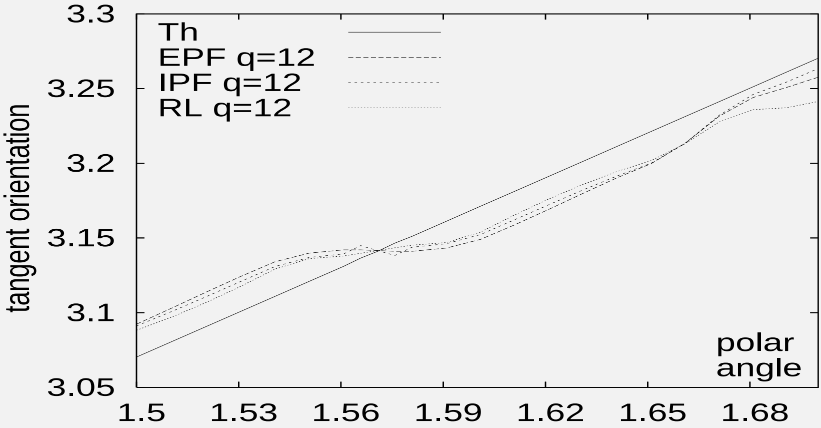
Polynomial Fitting : Tangent Orientation on Circle



radius 1, grid step = 0.01

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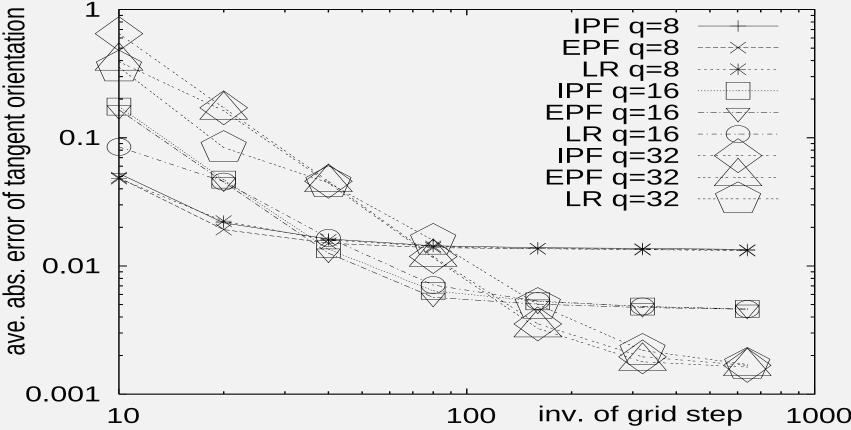
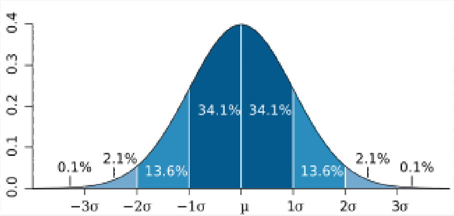


radius 1, 50 xp with random center shift

Gaussian Smoothing

- estimated derivative at C_0 obtained as : $\sum_{i=-q}^{q} G'_{\sigma_q}(-i)\mathbf{C}_i$
- $G'_\sigma(t)$, first derivative of $G_\sigma(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-t^2}{2\sigma^2}\right)$,

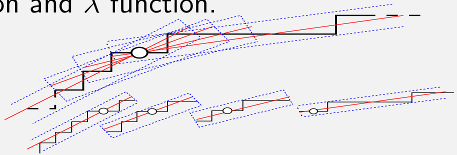


- $\sigma_q = \frac{2q+1}{3}$
- Estimation on whole curve in $\mathcal{O}\left((2q+1) * \text{number of points}\right)$.

Digital Estimator : λ-MST

Digital Straight Segment : piece of the digitization of a ray
Maximal DSS (MS for short) : non extensible DSS

- Weighted average of orientation of MS, weights depend on point location and λ function.



- Digital convexity $\equiv$ monotony of slopes [Doerksen et.al.'04]. Exists λ functions making no false concavities.
- Average length of maximal segments follows $\Theta(h^{-1/3})$: the higher the resolution, the more points inside max. DSS. [De Vieilleville et.al.'06]

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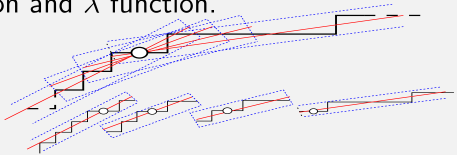

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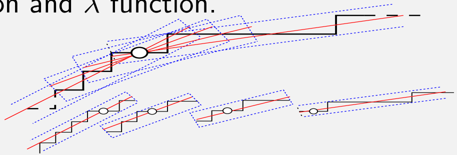

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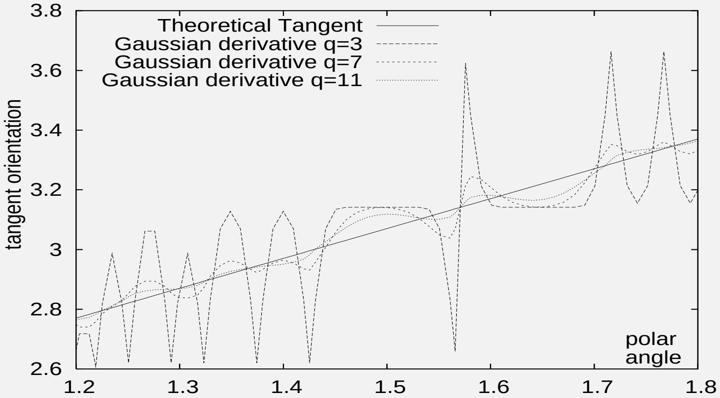
Defects of Continuous Methods : Digitized Circle (h=0.01)



→ Fixed computation window creates false concavity.

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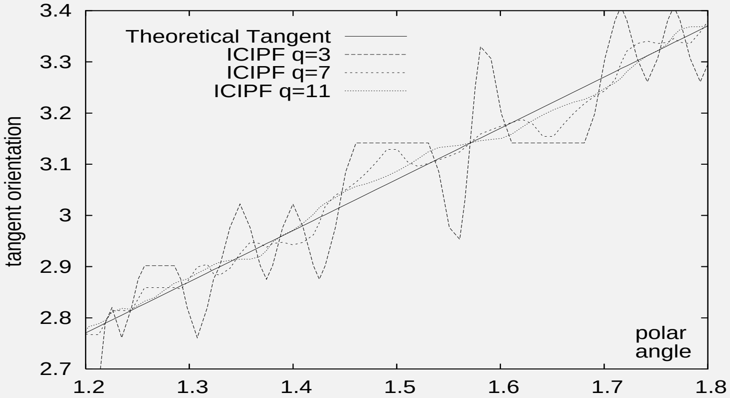

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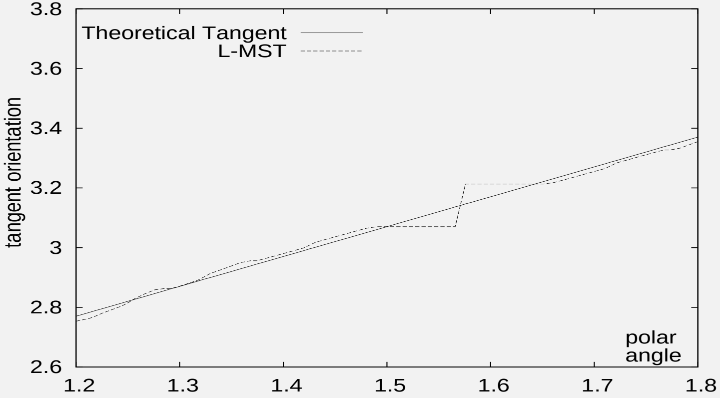
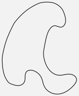
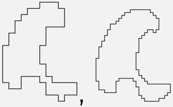


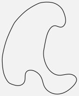
$\rightarrow$ λ-MST does not.

Single Criterion : Multi-Grid Convergence

Euclidean Object ($\mathcal{X}$)	Geometric Descriptor ($\mathcal{G}$)	Evaluation
	$\longrightarrow$	$\mathcal{G}(\mathcal{X})$
$\downarrow$ $DigG_h(\cdot)$ $\downarrow$		
 $, \ldots$	$\longrightarrow$	$\hat{\mathcal{G}}(DigG_h(\mathcal{X}))$
Digital Object ($DigG_h(\mathcal{X})$)	Estimation of Geometric Descriptor ($\hat{\mathcal{G}}$)	Estimation

Single Criterion : Multi-Grid Convergence

Euclidean Object ($\mathcal{X}$) | Geometric Descriptor ($\mathcal{G}$) | Evaluation

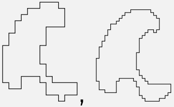


$\longrightarrow$ $\mathcal{G}(\mathcal{X})$

$\downarrow$

$DigG_h(\cdot)$ $\quad\uparrow\quad$ $\lim_{h \to 0}$? $\quad\uparrow$

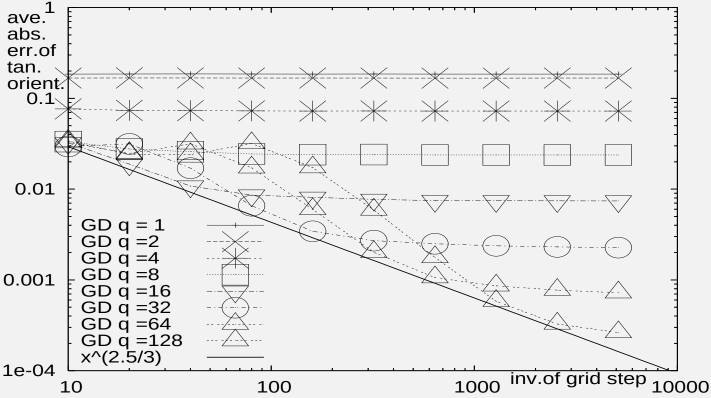
$\downarrow$



$, \quad , \ldots$ $\longrightarrow$ $\hat{\mathcal{G}}(DigG_h(\mathcal{X}))$

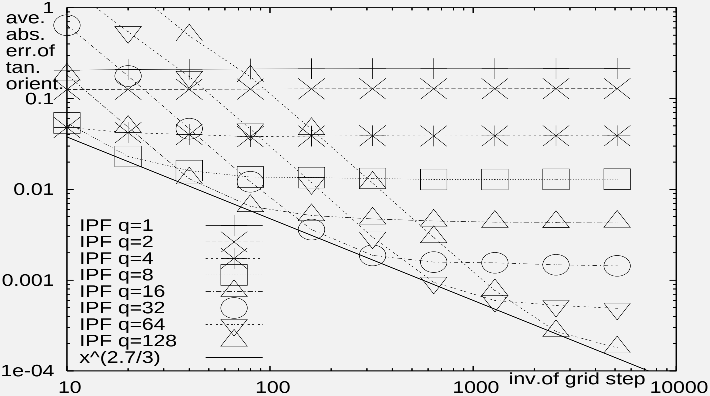
Digital Object ($DigG_h(\mathcal{X})$) | Estimation of Geometric Descriptor ($\hat{\mathcal{G}}$) | Estimation

Single Criterion : Multi-Grid Convergence



→ Fixed computation window prevents multi-grid convergence.

Single Criterion : Multi-Grid Convergence



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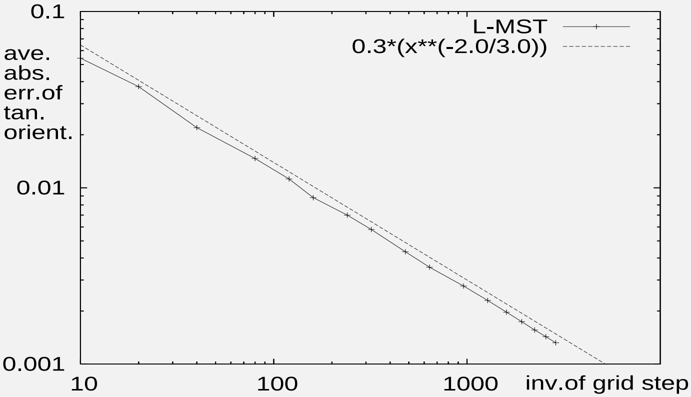
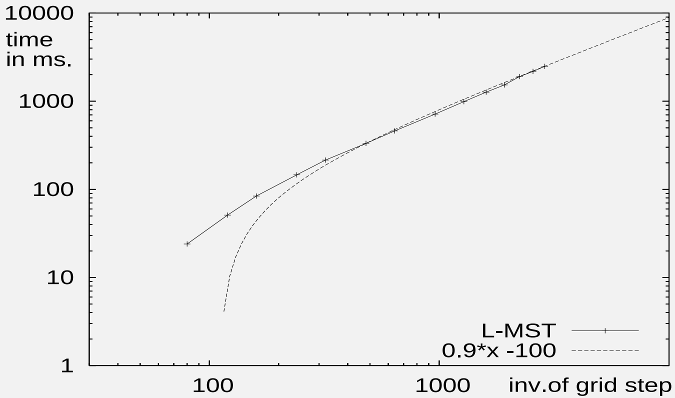


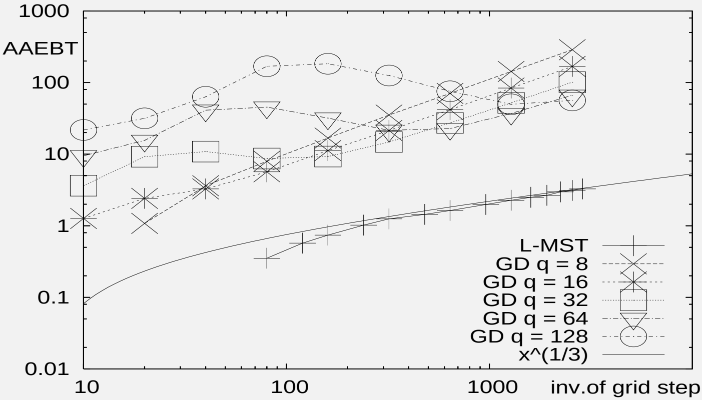
→ Adaptive computation window allows multi-grid convergence.

Single Criterion : Time



Linear wrt number of points of the digital curve, same order as GD and IPF.

Mixed Criterion : Average Abs. Error * Time



→ λ-MST is better... by far ! !

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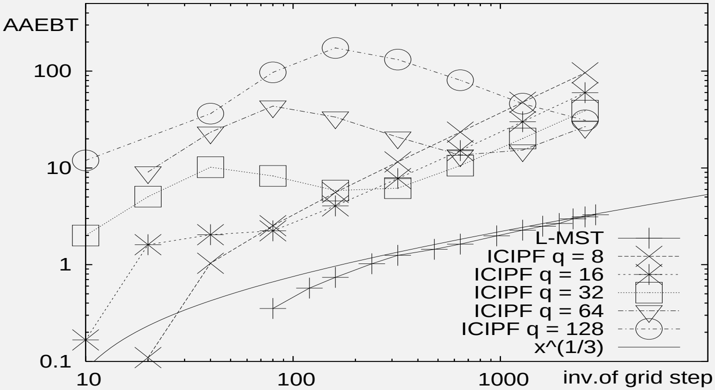

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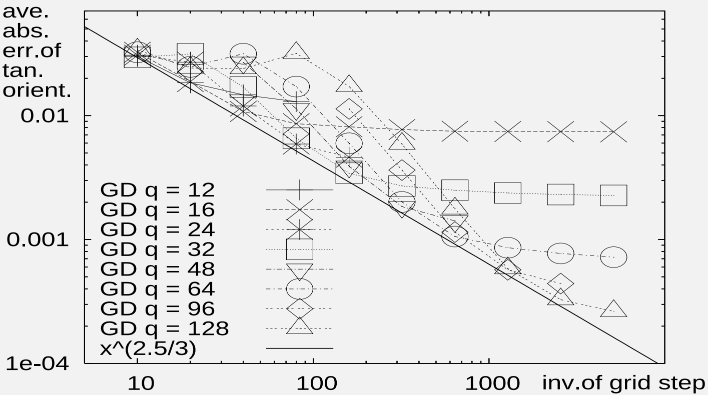
Best Computation Window Size : Experimental Suggestion



$\rightarrow$ Yell a convergence rate in $\mathcal{O}\left(h(-5/6)\right)$! !

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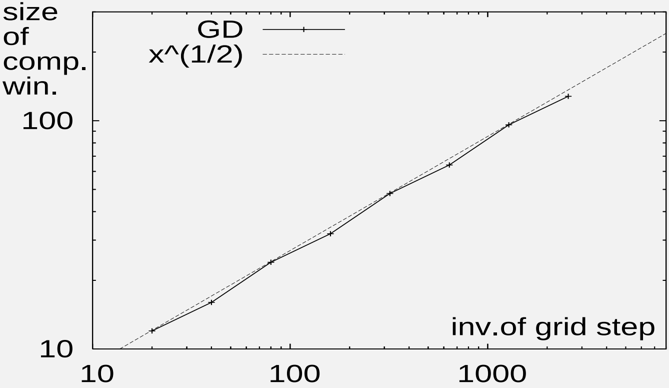
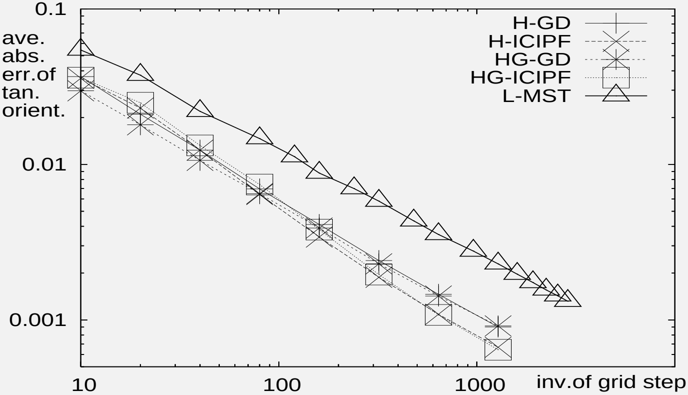


$\rightarrow$ Computation window size should grow in $\mathcal{O}\left(h(-1/2)\right)$

Computation Window Size : Based on MS ?



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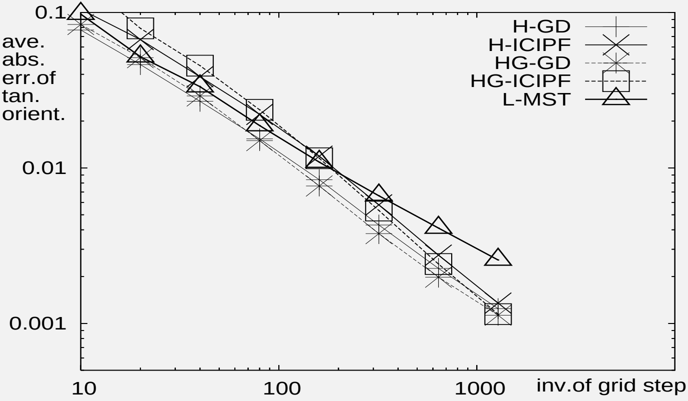

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- In the ideal case, λ-MST outperforms classic continuous methods,
- Continuous and digital methods can benefit from one another.

Future work will consider noise in evaluation.