Maximal Planes and Multiscale Tangential Cover of 3D Digital Objects

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Abstract. The sequence of maximal segments (i.e. the tangential cover) along a digital boundary is an essential tool for analyzing the geometry of two-dimensional digital shapes. The purpose of this paper is to define similar primitives for three-dimensional digital shapes, i.e. maximal planes defined over their boundary. We provide for them an unambiguous geometrical definition avoiding a simple greedy characterization as previous approaches. We further develop a multiscale theory of maximal planes. We show that these primitives are representative of the geometry of the digital object at different scales, even in the presence of noise.

1 Introduction

Maximal segments [5, 7] are the inextensible digital straight segments over the boundary of digital shapes. They have proven to be an essential tool for analyzing the geometry of 2D digital shapes, for instance for length and tangent estimation [14, 3], curvature estimation [7, 9], multiresolution analysis [6], unsupervised noise detection [10] or minimum length polygon computation. Furthermore, this theory of maximal segments can be extended to take into account possible noise in the data. The principle is simply to authorize thicker digital straight segments. This approach was proposed by Debled-Rennesson *et al.* [4], where a user specifies a maximal thickness in the segment decomposition.

Unfortunately, there is for now no equivalent theory for 3D or nD shapes, i.e. the definition and computation of maximal planes. A natural approach would be related to the convex hull, but it is not satisfactory for non convex shapes. Polyhedrization methods could also be considered as good candidates for defining maximal planes, since they are piecewise linear reconstruction of the shape boundary. Unfortunately, existing methods use greedy algorithms, whose result is for instance dependent on the starting point. Several polyhedrization methods [8, 12] starts from a point and greedily aggregates surrounding points as long as they form a digital planar set. Then, it chooses arbitrarily a new point and repeat the process until all points have been visited. As the reader may check on the freely available implementation of [16], these methods do not capture well linear or smoothly curved parts. More elaborate methods ([1] and especially [15]) address partially the problem of distinguishing polyhedral parts from smoothed parts. A user-given parameter and some *ad hoc* rules improve the previous polyhedrization algorithms. However, the obtained planes are again algorithmically obtained, without geometric or analytic description.

There exists a lot of reconstruction algorithms from scattered data in the image synthesis and computational geometry field. However, they do not take into account the specific geometry of digital spaces. For instance, if the object is the digital polyhedron, such algorithms cannot recognize digital planes as Euclidean planes. A staircase effect or an over-smoothing is generally the result of these algorithms.

The essential problem for defining interesting digital planes over a digital surface is that they must be characteristic of the local shape geometry, i.e. they should act as a local tangent plane. Now, no such problem exists in 2D, since the greedy inextensible digital linear sets — the so-called maximal segments — have been proven to be good tangent approximation [14]. However, in 3D, most inextensible digital planar sets are not characteristics of the local tangent geometry of the shape. For instance, any slice in the shape is planar, whatever the chosen direction. We must therefore find a way to select the representative planes within all these planar sets.

It is clear that the combinatorics of all planar sets of an object is too important. A natural approach is to find the smallest subset of these primitives which covers the digital boundary. But it is unlikely to find such an algorithm since this problem has been shown to be NP-complete [17].

We propose a new approach to tackle this problem by introducing the concept of *maximal (hyper)planes at a given scale* and the *hierarchy* of such primitives. Its objective is to satisfy the following requirements:

1. Maximal planes should approach the notion of tangent plane, whether the object under study is the digitization of a smooth shape with curvatures or the digitization of polyhedral surfaces.
2. It should take into account the specific nature of digital data, for instance, digital planes should be recognized in one piece.
3. Maximal planes must have a sound geometric definition. They are not only defined as the result of an algorithm, nor depend on user-given parameter(s).
4. They should be defined at different scales, so as to analyze the shape geometry at progressive resolutions. In this way, geometric analysis of noisy shapes can be addressed.
5. Ideally, this definition should encompass the classical 2D definition of maximal segments.

The paper is thus organized as follows. Maximal (hyper)planes are defined in Section 2, and their hierarchy is also presented. Section 3 proposes algorithms to compute them, a time complexity analysis is also carried out. Section 4 presents a natural application of hierarchical maximal planes: the estimation of the normal vector field of digital shapes. Results on known shapes show that maximal planes are characteristic of the shape geometry at different scales, whether the shape is the digitization of a polyhedral or smoothly curved object. Furthermore, the presence of noise is addressed by the multiscale hierarchy. Section 5 concludes and lists several future research perspectives.

2 Definition of Maximal Hyperplanes

In this section, we define the hierarchy of maximal planes that covers any discrete surface. We restrict our study to digital surfaces of $\mathbb{Z}^n$, but the framework remains valid for finite subsets of $\mathbb{R}^n$ with connectivity relations like triangulated surfaces.

2.1 Digital Surface and Tightiest Hyperplane

We recall that in the 3D cell complex approach of Kovalevsky (e.g., see [13, 11]), 3-cells (open unit cubes), 2-cells (open unit squares), 1-cells (open unit segments) and 0-cells (closed points) are respectively called voxels, surfels, linels and pointels.

The surface under study is always denoted by $\mathcal{S}$ and it corresponds to some graph (V, E), where the set of *vertices* V is a subset of $\mathbb{Z}^n$ and the set of *edges* E is the connectivity relation between these vertices. Note that this framework covers several definitions of digital surfaces, for instance:

- The set V may represent the border voxels of a digital object and E is then the neighborhood graph of V, choosing for instance the $(2n, 3^n - 1)$ adjacencies [Rosenfeld]
- The set V may represent the surfels centroid of a digital object boundary and E is then any bel adjacency [Herman93,Udupa94]
- The set V may represent the pointels of a digital object boundary and E is simply the grid 1-cells between them.

A subset X of $\mathbb{Z}^n$ is called *a piece of digital hyperplane* if it corresponds to the discretization of a piece of Euclidean hyperplane. As a Euclidean hyperplane is characterized by its normal vector, the same applies for digital hyperplanes.

We usually define arithmetically digital hyperplanes as follows. The set of points X in $\mathbb{Z}^n$ corresponds to a piece of *digital naive hyperplane* of normal vector $N = (N_1, \ldots, N_n) \in \mathbb{Z}^n$ if any point p of X satisfies:

$$\mu \leq N \cdot p < \mu + ||N||_\infty \tag{1}$$

where N is in its lowest terms. Let e_k denote the axis such that $||N||_\infty = N_k$, then e_k is called the *major axis* of the piece of digital hyperplane.

We can rewrite the preceeding double inequality as follows :

$$\frac{\max_{p \in X}(N \cdot p) - \min_{p \in X}(N \cdot p)}{||N||_\infty} < 1 \tag{2}$$

We can geometrically interpret this definition of digital naive hyperplane. Consider two Euclidean hyperplanes of normal vector N such that the set of points

X is contained between the two hyperplanes and such that at least one point of X lies one each hyperplane. These two hyperplanes are called *supporting hyperplanes* and (2) means that the Euclidean distance between these two hyperplanes relative to the major axis is strictly less than 1. We notice that, by allowing a larger distance between supporting hyperplanes, we can define thicker digital hyperplanes.

Obviously, a given subset X corresponds to the discretization of several Euclidean hyperplanes, resulting of a small variation of the normal vector. As a consequence, we choose the following definition, which has the advantage of defining without ambiguity one plane for a given subset X of $\mathbb{Z}^n$. Among all *arithmetic digital hyperplanes* which contain X, the one with the smallest axis-aligned thickness is called the *tightiest hyperplane* containing X, and is written $TH(X)$. If several axes induce a tightiest hyperplane, we choose the first one. The *normal vector* $N(X)$ to X is the normal vector to the tightiest hyperplane, pointing in the same half-space as its axis. The *thickness of* X is the thickness of $TH(X)$, otherwise said the quantity $\mathrm{t}(X) \stackrel{def}{=} \frac{\max_{p \in X}(N(X) \cdot p) - \min_{p \in X}(N(X) \cdot p)}{||N(X)||_\infty}$.

2.2 Neighborhood, ν-thick Disk and Extension

Let $\|\cdot\|$ be the Euclidean norm. The closed ball of radius r centered at some point p is denoted by $B_p(r)$. For a given vertex p of $\mathcal{S}$, the set of vertices of $\mathcal{S}$ lying in $B_p(r)$ defines a subgraph of $\mathcal{S}$. There may be several connected component in this subgraph, but the only one containing p is called the *r-neighborhood* of p in $\mathcal{S}$, and is written $\mathcal{S}_p(r)$.

The r-neighborhood of p admits a tightiest plane. Its thickness is clearly an increasing function of r, denoted by w_p. Since we are considering a finite graph, this function is piecewise constant on intervals $[r_k, r_{k+1}[$. Inversely, for a given thickness ν, the radius function $\rho_p(\nu)$ is defined as follows: $\rho_p(\nu) = r_k$ iff $\nu \in [w_p(r_k), w_p(r_{k+1})[$.

The subgraph $\mathcal{S}_p(\rho_p(\nu))$ is called the *ν-thick disk* around p, and is simply denoted with $\mathcal{S}_p^\nu$. The *ν-tightiest plane around* p is defined as $TH_p^\nu \stackrel{def}{=} TH(\mathcal{S}_p^\nu)$. Its normal direction is written as $\overrightarrow{N}_p^\nu$.

This plane is characteristic of the tangent space at p at a given scale ν. It forms a strip in the space of axis-thickness no greater than ν, which contains p and an isotropic connected neighborhood around it of radius $\rho_p(\nu)$. Having an isotropic neighborhood is interesting when analyzing the digitization of shapes with smooth boundaries. However, when the object under study contains planar facets (e.g. polyhedra, manufactured objects), this neighborhood is not always pertinent. It is therefore interesting to consider the extension to the largest connected component including p and belonging to TH_p^ν. The *ν-thick extension* around p is defined as the subgraph of $\mathcal{S}$:

$$\overline{\mathcal{S}}_p^\nu \stackrel{def}{=} \{\text{The connected component of } \{\mathcal{S} \cap TH_p^\nu\} \text{ that contains } p\}.$$

Since $\mathcal{S}_p^\nu \subset TH_p^\nu$, it is obvious that $\mathcal{S}_p^\nu \subset \overline{\mathcal{S}}_p^\nu$. The ν-thick extension thus contains the ν-thick disk around any vertex.

Figure 1 illustrates an example of a 1-thick disk (in yellow) and its surrounding ball (in green) on the surface of an ellipse (left). On the right, its 1-thick extension is represented.

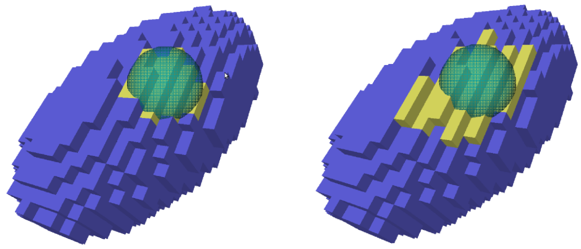


Fig. 1. A ν-thick disk and its surrounding ball (left), its ν-thick extension (right) ($\nu = 1$)

2.3 Maximal Disks and Hierarchy of Active Vertices

The preceding definitions may let us think that any point of a surface induces a maximal disk and a maximal plane for a given scale. This is rather counter-intuitive. We tend to think that the coarser is the scale the simpler is the object. We would expect less maximal disks and extensions at coarser scales. We therefore introduce a mechanism to keep only the most significant disks and extensions at each scale. Furthermore, this mechanism naturally induces a hierarchy on these neighborhoods.

A ν-thick disk is *maximal* iff for any vertex $q \in \mathcal{S}$, $q \neq p$, the ball $B_p(\rho_p(\nu))$ is not included in the ball $B_q(\rho_q(\nu))$ or $p \notin TH_q^\nu$. The first condition guarantees that $\mathcal{S}_p^\nu \subset \mathcal{S}_q^\nu$ whenever p and q lie in the same component of $\mathcal{S} \cap B_p(\rho_q(\nu))$. The second condition approximates this connection condition efficiently.

We may now define a hierarchy of *active vertices* for any sequence of increasing thicknesses $(\nu_i)_{i=0,\ldots,L}$, with $\nu_0 = -1$, $\nu_1 = 0$, $\nu_L = \mathrm{t}(\mathcal{S})$. For the sake of convenience, we admit that the first thickness of the sequence is negative. The following process defines a hierarchy $(\mathcal{F}^{\nu_i})_{i=-1,\ldots,L}$:

$$\mathcal{F}^{\nu_i} = \begin{cases} \{\text{set of vertices of } \mathcal{S}\} \text{ if } i = 0, \\ \{p \in \mathcal{F}^{\nu_{i-1}} : \mathcal{S}_p^{\nu_i} \text{ is maximal}\} \text{ if } i \geq 1. \end{cases} \tag{3}$$

Furthermore, if the sequence (ν_i) is refined uniformly, then the family of sets converges uniformly toward a piecewise constant family of sets $(\mathcal{F}^\nu)_{\nu \in \mathbb{R}}$, with the convenient convention that $\forall \nu < 0, \mathcal{F}^\nu = \mathcal{S}$. The sequence of values ν, such that $\mathcal{F}^\nu$ is only right-continuous, represents the *stable scales for* $\mathcal{S}$ and are denoted as a sequence $(\mu_i)_{i=0,\ldots,L'}$. By construction, this hierarchy of active vertices at progressive scales only depends on the input surface $\mathcal{S}$ (vertices and edges).

By definition of the hierarchy $(\mathcal{F}^{\nu_i})_{i=-1,\dots,L}$, it is obvious that $\mathcal{S} = \mathcal{F}^{<0} \supset \mathcal{F}^0 \supset \dots \supset \mathcal{F}^{\mu_i} \supset \mathcal{F}^{\mu_{i+1}} \supset \dots \supset \mathcal{F}^{\mu_{L'}}$. As a consequence, we obtain the following property:

Property 1. $\forall \mu, \mu' \in \mathbb{R}, \mu < \mu' \Rightarrow \mathcal{F}^{\mu} \supseteq \mathcal{F}^{\mu'}$.

Moreover, we notice that at a given scale ν each vertex $q \in S$ belongs to at least one ν-thick disk. This leads to the next property :

Property 2. $\forall \nu \in \mathbb{R}$, $\bigcup_{p \in \mathcal{F}^{\nu}} \mathcal{S}_p^{\nu} = \mathcal{S}$ and $\cup_{p \in \mathcal{F}^{\nu}} \overline{\mathcal{S}}_p^{\nu} = \mathcal{S}$.

Definition 1. *The* tangential cover $\mathbb{T}^{\nu}$ at scale ν of $\mathcal{S}$ *is the set of subgraphs*

$$\mathbb{T}^{\nu} \stackrel{def}{=} \{\overline{\mathcal{S}}_p^{\nu} : p \in \mathcal{F}^{\nu}\}.$$

Each element of $\mathbb{T}^{\nu}$ *is called a* ν-maximal plane of $\mathcal{S}$. *The set of* ν*-maximal planes containing a vertex* p *is called the* ν-linear pencil *of* p.

Figure 2 shows an example of maximal 1-thick disk (yellow) containing two different non maximal 1-thick disk.

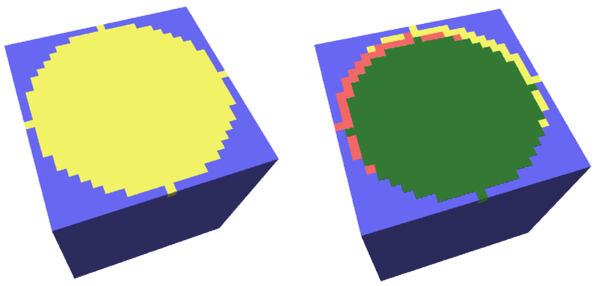
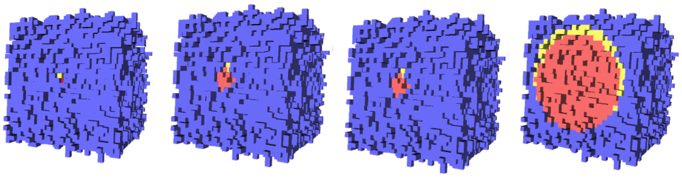


Fig. 2. 1-thick maximal disk in yellow (left), 1-thick disks included in the yellow one (right)

Figure 3 shows an example of two maximal ν-thick disks in red and yellow for $1 \le \nu \le 3$ (from (a) to (c)) on a noisy surface. The red one is not maximal anymore when $\nu = 4$ because it is included in the yellow one, as a consequence, its center is not an active vertex anymore at scale 4.

3 Computation and Time Complexity

In this section, we present our algorithm for the computation of the hierarchy of active vertices. This hierarchy leads to the computation of the tangential cover of the surface at different scales.



(a) $\nu = 1$ (b) $\nu = 2$ (c) $\nu = 3$ (d) $\nu = 4$

Fig. 3. Two distinct maximal ν-thick disks for $1 \leq \nu \leq 3$, inclusion of the red 4-thick disk in the maximal yellow one

3.1 Algorithm Design

Our method computes the hierarchy of active vertices $\mathcal{F}^{\nu_i}_{i=-1,\ldots,K}$ for a given sequence of increasing thicknesses $(\nu_i)_{i=0,\ldots,K}$, with $\nu_0 = -1$, $\nu_1 = 0, \ldots$, relative to a given surface $\mathcal{S}$. It is summarized in Algorithm 1 and we return to the main steps in the following.

Algorithm 1: Hierarchy of active vertices algorithm

INPUT : $\mathcal{S}$, $(\nu_i)_{i=0,\ldots,K}$
OUTPUT : $\mathcal{F}^{\nu_i}_{i=-1,\ldots,K}$
    $Q \leftarrow NULL$
    $\mathcal{F}^{-1} \leftarrow$ {set of vertices of $\mathcal{S}$}
    FOR EACH increasing thickness ν_i, $0 \leq i \leq K$
        $L \leftarrow NULL$
        FOR EACH vertex p of $\mathcal{F}^{\nu_{i-1}}$
            Q.push($\mathcal{S}^{\nu_i}_p$);
        WHILE(!empty(Q))
            $A \leftarrow Q$.pop()
            IF($\forall B \in L$, !($A \subset B$))
                L.push(A)
                $\mathcal{F}^{\nu_i} \leftarrow \mathcal{F}^{\nu_i} \cup A$.center()

By convention, $\mathcal{F}^{-1}$ corresponds to the set of vertices of $\mathcal{S}$. For each thickness ν_i, $0 \leq i \leq K$, the set $\mathcal{F}^{\nu_i}$ is induced by the set $\mathcal{F}^{\nu_{i-1}}$ and the main steps of our approach are the following :

1. Computing the ν_i-thick disk around p, for each vertex p belonging to $\mathcal{F}^{\nu_{i-1}}$. The ν_i-thick disks are stored in a priority queue (denoted by Q) ordered relative to the radius of the disks, the largest disk lying on the top of the priority queue.
2. Extracting of the priority queue the maximal ν_i-thick disks and adding their vertex center to $\mathcal{F}^{\nu_i}$

The first step consists in computing the ν_i-thick disk $\mathcal{S}_p^{\nu_i}$ around p, for each vertex p belonging to $\mathcal{F}^{\nu_{i-1}}$. For this, we use in an incremental way a digital plane recognition algorithm such as the COBA algorithm introduced in [2]. The method adds neighboors of p in a specific order as long as they belong to the same ν_i-thick plane. Let us call r_k*-ring* of a point p the subgraph defined by $\mathcal{S}_p(r_k) \setminus \mathcal{S}_p(r_{k-1})$. It adds the vertices of the r_k-ring of p, for each increasing $k > 1$. If a vertex of the r_k-ring cannot be added (the set of vertices will not correspond to a ν_i-thick disk anymore), all the vertices of r_k-ring of p are removed from the subgraph and the algorithm stops. We illustrate the construction of ν_i-thick disk around p, denoted by $\mathcal{S}_p^{\nu_i}$, in Algorithm 2. We use a priority queue, denoted by Q, which contains vertices ordered relative to their Euclidian distance to the initial point p. The closest vertex lies on the top of the queue.

Algorithm 2: The ν-thick disk algorithm

INPUT : $\mathcal{S}$, ν, p
OUTPUT : $\mathcal{S}_p^{\nu}$
    $DP \leftarrow TRUE \quad \mathcal{S}_p^{\nu} \leftarrow \emptyset \quad Q \leftarrow NULL$
    Q.push(p)
    WHILE(!empty(Q) AND DP)
        $q \leftarrow Q$.pop()
        IF($q \notin \mathcal{S}_p^{\nu}$)
            $\mathcal{S}_p^{\nu} \leftarrow \mathcal{S}_p^{\nu} \cup q$
            IF(isDigitalPlane($\mathcal{S}_p^{\nu}$,ν))
                Q.push(neighboors of q)
            ELSE
                $r \leftarrow q$.EuclidianDistanceTo(p)
                $\mathcal{S}_p^{\nu} \leftarrow \mathcal{S}_p^{\nu} \setminus r$-ring($p$)
                $DP \leftarrow FALSE$

The second step consists in extracting the maximal ν_i-thick disks and adding their vertex center to $\mathcal{F}^{\nu_i}$. At the end of the first step, ν_i-thick disks are stored in a priority queue ordered relative to their radius. We remove each disk of the priority queue, beginning with the disk on the top, and we wonder whether it is maximal or not. If it is maximal, it is added to a queue called L and its center is added to $\mathcal{F}^{\nu_i}$. Obviously, a disk cannot be included in one of smaller radius. We also notice that, because two disks cannot have the same center, a disk cannot be included in one of the same radius. The first extracted disk is obviously maximal and so it is added to the queue L. By construction of the priority queue, for each extracted disk d, we only have to test the inclusion of d in the disks of the queue L.

3.2 Time Complexity Analysis

We propose to analyse the time complexity of the hierarchy of active vertices algorithm (Algorithm 1).

Let $\mathcal{S}$ denote the surface under study, let us denote by D and m its diameter and the number of vertices of its associated subgraph respectively. According to Property 1, for all i such that $0 \leq i \leq k$, $\mathcal{F}^{\nu_i}$ is included in or equal to $\mathcal{F}^{\nu_{i-1}}$. As a consequence, the number of vertices of each $\mathcal{F}^{\nu_i}$ is bounded by m, for all i such that $-1 \leq i \leq k$. As the time complexity of the computation of each $\mathcal{F}^{\nu_i}$ only depends on the cardinality of its ascending set and of the diameter of the surface, we can study separately the time complexity of the computation of a set $\mathcal{F}^{\nu_i}$ whatever ν_i.

For a given thickness ν_i, the first step consists in computing the maximal ν_i-thick disks centered on each vertex of the set $\mathcal{F}^{\nu_{i-1}}$ and adding it to priority queue. To compute each ν_i-thick disk, the COBA algorithm is used to incrementally construct a maximal isotropic piece of digital plane of thickness ν_i. As in the non-incremental case, the COBA algorithm runs in $O(m \log(D))$ time (see [2]). The pushing operation on a prority queue runs in $O(\log(l))$ time where l denotes the size of the priority queue. As the size of our priority queue is obviously bounded by m, this step runs in $O(\log(m))$ time. Because $O(\log(m))$ can be neglected relative to $O(m \log(D))$, the first step runs in $O(m^2 \log(D))$ time.

The second step consists in extracting the maximal ν_i-thick disks. For each ν_i-thick disk of the priority queue, the method tests whether it is included in a larger one. As the number of larger ν_i-thick disks is bounded by m for each disk, deciding whether it is maximal or not runs in $O(m)$ time. As a consequence, checking the maximality of every disks of the priority queue runs in $O(m^2)$ time.

To conclude, as $O(m^2)$ can be neglected relative to $O(m^2 \log(D))$, the computation of the set $\mathcal{F}^{\nu_i}$ for a given thickness ν_i runs in $O(m^2 \log(D))$ time in the worst case. The computation of all the hierarchy of active vertices for $(\nu_i)_{i=0,\dots,L}$ runs in $O(Lm^2 \log(D))$ in the worst case.

4 Application

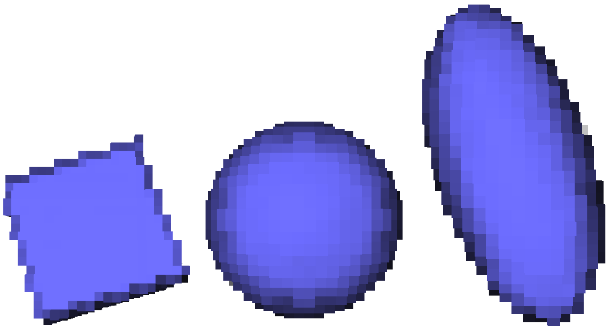
We proposed in Section 3 an algorithm to compute the hierarchy of active vertices at different scales. Moreover, we introduced the definition of the tangential cover of a surface $\mathcal{S}$ at a given scale ν (see Definition 1), induced by the set of active vertices at scale ν.

Knowing the tangential cover $\mathbb{T}^\nu$ of a surface $\mathcal{S}$ at a scale ν naturally leads to the normal estimation of the surface at each point at this scale. Indeed, each ν-maximal plane of $\mathbb{T}^\nu$ represents a tangent plane for each covered vertex. To each vertex v of the surface, we associate the average of the normal vectors of each ν-maximal plane of its ν-linear pencil.

For a regular discrete surface, we expect the normal estimation to be well representative at scale 1. Conversely, if the discrete surface is noisy, the normal estimation could be wrong for some vertices representing noise at scale 1. Nevertheless, we expect that at coarser scales our normal estimation gets improved, until we reach the stable scale. This stable scale represents the global noise level added on the surface.

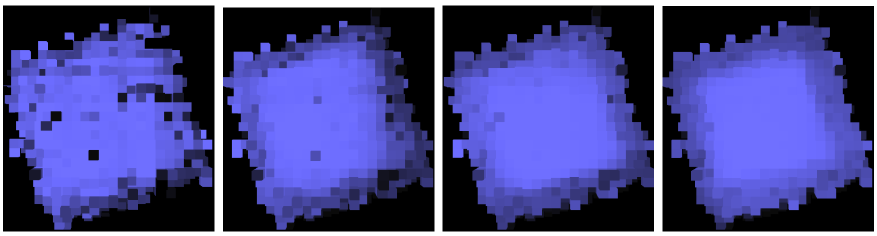
We implement our method in C++ using the ImaGene library. We generate three-dimensional regular discrete shapes, we choose to add noise or not, we compute its tangential cover at a given scale and we estimate a normal vector at each surface voxel. In order to visualise the result, we display the three-dimensionnal surface using Inventor by associating to each surface voxel its estimated normal vector. As a consequence, when the surface is lighted, we can immediatly judge whether the normal estimation is consistent.

Figure 4 shows the result of our normal estimation at scale 1 on the regular surface of a cube, of a sphere and of an ellipse. Figure 5 and figure 6 show the result of our normal estimation on noisy surfaces of a cube and of an ellipse respectively, at scales 1, 2 ,3 and 4. We notice that the normal estimation gets more and more consistent until stability at scale 4.



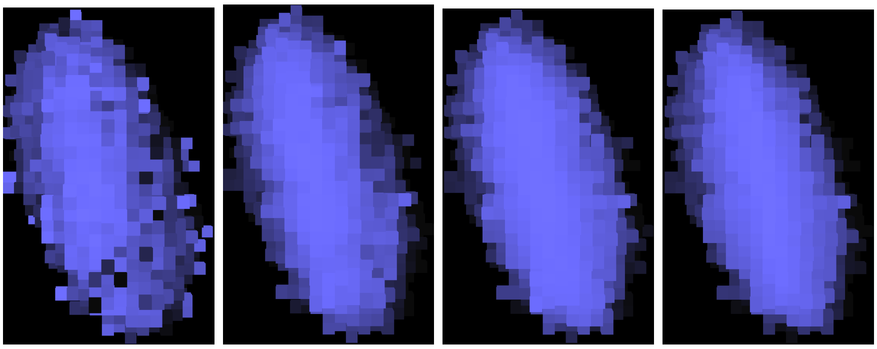
(a) (b) (c)

Fig. 4. Normal estimation at scale 1 of regular discrete surfaces



(a) $\nu = 1$ (b) $\nu = 2$ (c) $\nu = 3$ (d) $\nu = 4$

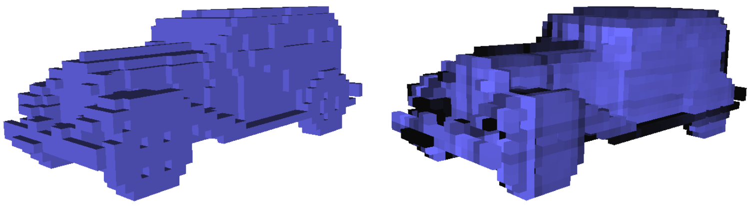
Fig. 5. Normal estimation at scale 1, 2, 3 and 4 of a noisy discrete surface : a cube



(a) $\nu = 1$ (b) $\nu = 2$ (c) $\nu = 3$ (d) $\nu = 4$

Fig. 6. Normal estimation at scale 1, 2, 3 and 4 of a noisy discrete surface : an ellipse

Finally, we show the result of our normal estimation on realistic data representing an old car (data available on the TC18 website[3]). Figure 7 shows the original object and the result of our normal estimation at scale 1. Figure 8 shows the original object with additional noise and the result of our normal estimation on this noisified object at scale 1, 2 and 3.



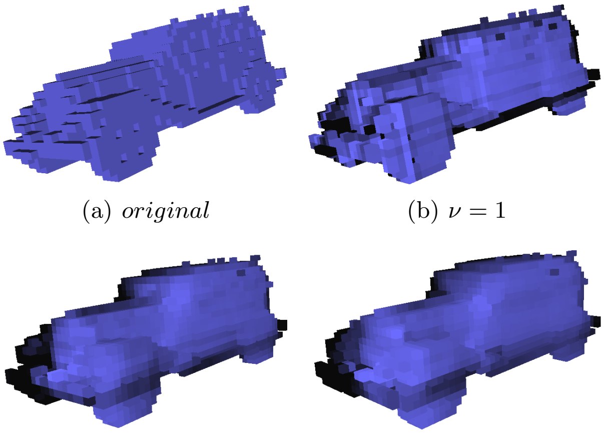
(a) (b)

Fig. 7. Discrete object representing a Dodge (left), normal estimation at scale 1 (right)

5 Conclusion and Perspectives

We propose in this paper a new definition for the tangential cover of a three-dimensional digital shape based on the computation of maximal planes over its surface. Moreover, we provide an algorithm to compute the hierarchy of active vertices at different scales, which induces the tangential cover of the shape at each scale. We highlight the fact that maximal planes are locally representative of the shape and so that they can be used to estimate the normal vector at each point

[3] http://tc18.liris.cnrs.fr



(a) *original* (b) $\nu = 1$

(c) $\nu = 2$ (d) $\nu = 3$

Fig. 8. Original object and normal estimation at scale 1, 2 and 3 of a noisy discrete object : a dodge

on the surface. The study of maximal planes over a discrete surface could also lead to estimation of local or global noise level on a three-dimensional surface. Moreover, we think that, as they provide an estimation of the normal vector at each point, they could also be used to extract other geometric characteristics as the curvature. We could work on them to discriminate curve parts from flat parts on the surface or to divide up the surface in convex and in concave parts.

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